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P. B. Wilcox.

# KEY

TO THE

# COMPLETE SYSTEM

OF

# MENSURATION OF SUPERFICIES AND SOLIDS,

CONTAINING

SOLUTIONS TO ALL THE PROBLEMS AND

QUESTIONS THEREIN CONTAINED,

CALCULATED FOR THE USE OF SCHOOLS, ACADEMIES,
AND PRIVATE LEARNERS.

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"MATHEMATICAL EXPOSITOR," &C.

NEW YORK:
PUBLISHED BY M'ELRATH, BANGS & Co.
STEREOTYPED BY J. S. REDFIELD.
1834.

Hist. of Sci
Blackfriar's
10-7-30
22655

03-19-34 MW

# CONTENTS.

# KEY

TO THE

# COMPLETE SYSTEM OF MENSURATION.

## SECTION I.

### OF THE SQUARE.

#### PROBLEM I.

EXAMPLES.

1. $20 \times 20 = 400$ inches.
2. $10 \times 10 \times 30,25 \times 9 = 27225$ square feet.
3. $60 \times 60 = 3600$ and $160)3600( = 22,5$ acres.
4. $160 \times 5 \times 30,25 = 24200$ men.
5. $14 \times 14 = 196$ square chains, and $100 \times 100 = 10000$ square links in one chain; then $196 \times 10000 = 1960000$ square links.

#### PROBLEM II.

EXAMPLES.

1. $\sqrt[2]{625} = 25$ feet.
2. $\sqrt[2]{384} = 19,5959 +$.
3. $160 \times 6 = \sqrt{960} = 30,9838 +$.
4. $125 \times 10 = 1250$ chains, and $\sqrt[2]{1250} = 35,3553 +$.

#### PROBLEM III.

EXAMPLES.

1. $16 \times 16 = 256$, and $2)256 = 128$ the area in chains, and $10)128 = 12,8$ acres.
2. $41,3 \times 41,3 = 1705, 69$ the square of the diagonal, and $2)1705,69 = 852,845$ rods area of the square,

and 160)852,845 = 5,33028 acres = to 5 acres, 1 rood, and 13 + rods.

3. 4,78 × 4,78 = 22,8484, and 2)22,8484 = 11,4242 square chains, and dividing by 10, because 10 square chains make an acre, = 1,14242, which is = to 1 acre and 22 + perches.

### PROBLEM IV.

#### EXAMPLES.

1. 64,8 × 10 = 648 chains the area, and 648 × 2 = 1296 and $\sqrt[2]{1296}$ = 36 chains the diagonal.

2. 4 = here 578 chains × 4 = 2312 rods, and 2312 × 2 = 4624 rods double the area, and $\sqrt[2]{4624}$ = 68 rods, length of the diagonal.

### PROBLEM V.

#### EXAMPLES.

1. 50 × 50 = 2500 chains the square of the diagonal; then 2500 divided by 2 = 1250 chains, and $\sqrt[2]{1250}$ = 35,3553 + chains, length of the side required.

2. 24 × 24 = 576 the square of the diagonal, and 2)576 = 288, and $\sqrt[2]{288}$ = 16,97 + rods.

3. 36 × 36 = 1296, and 2)1296 = 648, and $\sqrt[2]{648}$ = 25,4558 +.

### PROBLEM VI.

#### EXAMPLES.

1. 19 × 19 × 2 = 722, and $\sqrt[2]{722}$ = 26,87 + chains, length of the diagonal required.

2. 24 × 24 = 576 the square of one side, and 576 × 2 = 1152, and $\sqrt[2]{1152}$ = 33,9411 + rods, length of the diagonal.

3. 43 × 43 × 2 = 3698, and $\sqrt[2]{3698}$ = 60,8111 + feet.

### PROBLEM VII.

#### EXAMPLES.

1. 20 × 20 × 2 = 800 and $\sqrt[2]{800}$ = 28,28427, and 28,28427 + 20 = 48,28427 chains length of the side,

and 48,28427 × 48,28427 = 2331,37 square chains, which is = to 233 acres 21,92 rods.

2. 12 × 12 = 144 × 2 = 288, and $\sqrt[2]{288}$ = 16,97, and 16,97 + 12 = 28,97 rods length of the side, and 28,97 × 28,97 = 839,2609 square rods, and 160)839,-2609 = 5,245 acres = to 5 acres and 39 rods.

3. 8 × 8 × 2 = 128, and $\sqrt[2]{128}$ = 11,3136 +, and 11,3136 + 8 = 19,3136, and 19,3136 × 19,3136 = 373 + feet, the area.

## OF PARALLELOGRAMS.

### PROBLEM I.

#### EXAMPLES.

1. 24 × 16 = 384 poles area required.

2. 36 × 18 = 648 chains, and divided by 10 = 64,8 acres.

3. 14 × 15 = 210, and 12)210 = 17,5 feet.

4. 16 × 19 × 20 = 6080, and 12)6080 = 506 feet 96 square inches, = to $\frac{2}{3}$ of a square foot.

5. 36 × 18 = 648 square feet, and 100)648 = 6,48 squares.

### PROBLEM II.

#### EXAMPLES.

1. 30)456 = 15 chains and 20 links.

2. 12)784 = 65,3333 feet, or 65$\frac{1}{3}$ feet.

3. 36 × 4 + 3 = 147, and 147 × 40 + 30 = 5910 perches, and 120)5910 = 49,25 perches, length of the shortest side.

### PROBLEM III.

#### EXAMPLES.

1. 2)60 = 30, and 30 × 30 = 900, and 900 — 576 = 324, and $\sqrt[2]{324}$ = 18, and 30 + 18 = 48 chains the longest side, and 30 — 18 = 12 chains the shorter.

2. 3 × 4 + 1 = 13 roods, and 13 × 40 + 32 = 552 rods the area, and 116 rods = twice the sum of the length and breadth; therefore 4)116 = 29, and 29 × 29 = 841, and 841 — 552 = 289, and $\sqrt[2]{289}$ = 17, and

$29 + 17 = 46$ rods length of the longest side, and $29 - 17 = 12$ rods length of the shorter side.

3. $20 + 8 = 28$, and $2)28 = 14$, and $14 \times 14 = 196$, and $20 \times 8 = 160$, and $8)160 = 20$, and $196 - 20 = 176$, and $\sqrt[2]{176} = 13,2665$, and $14 - 13,2665 = ,7335$ — rods, width of the walk.

### PROBLEM IV.

#### EXAMPLES.

1. $4 \times 4 \times 40 = 640$ rods area, and $2)12 = 6$ half the difference of the two sides, and $6 \times 6 = 36$, and $640 + 36 = 676$, and $\sqrt[2]{676} = 26$ half the sum of the length and breadth; then $26 + 6 = 32$ rods the longer side, and $26 - 6 = 20$, shorter side.

2. $138 \times 10 = 1380$ chains the area, and $2)16 = 8$ half the difference of the two sides, and $8 \times 8 = 64$, and $1380 + 64 = 1444$, and $\sqrt[2]{1444} = 38$ half the sum of the two sides; then $38 + 8 = 46$ chains the longer of the two sides, and $38 - 8 = 30$ chains, the shorter side.

### PROBLEM V.

#### EXAMPLES.

1. $40 \times 4 + 3 = 163$ roods, and $163 \times 40 + 24 = 6544$ rods the area, and $6544 \times 3 = 19632$, and $4)19632 = 4908$, and $\sqrt[2]{4908} = 70,057$ rods the shorter side, and $70,057 \times 4 = 280,228$, and $3)280,228 = 93,4093$, length of the longer side.

2. $24 \times 10 = 240$ chains area; then $240 \times 2 = 480$, and $3)480 = 160$, and $\sqrt{160} = 12,6491 +$ chains the shorter side, and $12,6491 \times 3 = 38,0473$, and $2)38,0473 = 19,02365$ chains the longer side.

### PROBLEM VI.

#### QUESTIONS.

1. $12)720 = 60$ rods the sum of the adjacent sides; then $2)60 = 30$, and $2)12 = 6$; then $30 + 6 = 36$ rods the longest side, and $30 - 6 = 24$ rods the shortest.

2. 6)108 = 18 chains, sum of the two adjacent sides, and 2)18 = 9, and 2)6 = 3; then 9 + 3 = 12 chains the longer side, and 9 — 3 = 6 the shorter.

### PROBLEM VII.

#### EXAMPLES.

1. 20)80 = 4 chains, the difference between the sides then 2)20 = 10 chains the half sum, and 2)4 chain half the difference; then 10 + 2 = 12 chains longes side, and 10 — 2 = 8 the shortest.

2. 16)32 = 2 feet, difference of the 2 sides; then 2)2 = 1, half the difference, and 2)16 = 8 feet half the sum then 8 + 1 = 9 feet longest side, and 8 — 1 = 7 feet the shortest.

### PROBLEM VIII.

#### EXAMPLES.

1. 8 × 8 = 64 chains, square of their difference, and 194 the sum of their squares; then 194 — 64 = 130, and 130 + 194 = 324, and √324 = 18 sum of the two sides; then 2)18 = 9 the half sum, and 2)8 = 4 half their difference; then 9 + 4 = 13 chains the longest side, and 9 — 4 = 5 chains the shortest side.

2. 11 × 11 = 121 the square of the difference; then 281 — 121 = 160, and 281 + 160 = 441, and √441 = 21 inches the sum of the two sides; then 2)21 = 10,5 the half sum, and 2)11 = 5,5 half the difference; then 10,5 + 5,5 = 16 inches the longest side, and 10,5 — 5,5 = 5 inches the shortest side.

### PROBLEM IX.

#### EXAMPLES.

1. 50 × 50 = 2500 the square of their sum; then 2500 — 1300 = 1200, and 1300 — 1200 = 100 the square of the difference of the two sides; then √100 = 10 the difference; then 2)50 = 25 the half sum, and 2)10 = 5 half the difference; therefore 25 + 5 = 30 chains the longest, and 25 — 5 = 20 the shortest side;

then $30 \times 20 = 600$ chains the area, and $10)600 = 60$ acres the area required.

2. $2)56 = 28$ rods, the sum of the length and breadth of the garden; then $28 \times 28 = 784$ the square of their sum; then $784 - 464 = 320$, and $464 - 320 = 144 =$ the difference of the squares of the two adjacent sides, and $\sqrt{144} = 12$ the difference; then $2)28 = 14$ the half sum, and $2)12 = 6$ half the difference; therefore $14 + 6 = 20$ rods the longest sides, and $14 - 6 = 8$ rods the shortest, and $20 \times 8 = 160$ rods $=$ to one acre, the area required.

### PROBLEM X.

#### EXAMPLES.

1. $30 \times 10 = 300$ square chains the area, and $300 \times 2 = 600$ twice the area, and $25 \times 25 = 625$ chains the square of the diagonal, and $625 - 600 = 25$ difference of the squares of the two adjacent sides; then $\sqrt{25} = 5$ chains, difference of the two sides; then $625 + 600 = 1225$ chains, square of the sum of the two sides; consequently $\sqrt{1225} = 35$ chains, the sum of the two sides; then $2)35 = 17,5$ the half sum, and $2)5 = 2,5$ half their difference; then $17,5 + 2,5 = 20$ chains the longest side, and $17,5 - 2,5 = 15$ chains, the shortest side.

2. $6 \times 4 = 24$ roods, and $24 \times 40 + 12 = 972$ rods, the area; then $972 \times 2 = 1944$ rods twice the area, and $45 \times 45 = 2025$, and $2025 - 1944 = 81$ the square of the difference of the two sides; then $\sqrt{81} = 9$ rods the difference, and $2025 + 1944 = 3969$ the square of the sum of the two sides; then $\sqrt{3969} = 63$ rods, the sum of the two sides; then $2)63 = 31,5$ rods the half sum, and $2)9 = 4,5$ rods, half the difference; then $31,5 + 4,5 = 36$ rods the longest side, and $31,5 - 4,5 = 27$ rods the shortest.

### PROBLEM XI.

#### EXAMPLES.

1. 4 acres, 3 roods, and 8 rods $= 768$ rods the area; then $16)768$ rods $= 48$ chains the area, and $48 \times 48 =$

2304 the square of the area, and 2)28 = 14 half the difference of their squares; then 14 × 14 = 196, and 2304 + 196 = 2500, and $\sqrt{2500}$ = 50, and 50 + 14 = 64, the square of the longest side; then $\sqrt{64}$ = 8 chains the longest side, and 50 — 14 = 36 the square of the shortest; then $\sqrt{36}$ = 6 chains the shortest side.

2. 48 × 48 = 2304 the square of the area; and 2)128 = 64 half the difference of their squares; then 64 × 64 = 4096, and 4096 + 2304 = 6400, and $\sqrt{6400}$ = 80, and 80 + 64 = 144 the square of the longest side; then $\sqrt{144}$ = 12 chains the longest side, and 80 — 64 = 16 the square of the shortest; then $\sqrt{16}$ = 4 chains the shortest side.

## OF THE RHOMBUS.

### PROBLEM I.

#### EXAMPLES.

1. 12,24 × 9,16 = 112,1184 feet, the area required.

2. 12 feet 6 inches = 12,5 feet and 9 feet 3 inches = 9,25 feet; then 9,25 × 12,5 = 115,625 feet, the answer required.

### PROBLEM II.

#### EXAMPLES.

1. 5)24 = 4,8 rods, the perpendicular required.

2. 8 feet 6 inches are = to 8,5 feet; then 8,5)125 = 14,7 + feet, length of the side.

## OF THE RHOMBOID.

### PROBLEM I.

#### EXAMPLES.

1. 26 × 8 = 208 rods = to 1 acre, 1 rood, and 8 rods.

2. 18 feet 9 inches = 18,75 feet, and 12 feet 3 inches = 12,25 feet; then 18,75 × 12,25 = 229,6875 square feet; then 9)229,6875 = 25,5208 + square yards.

3. 24,75 × 36 = 891 square feet; then 9)891 = 99 square yards; then 30,25)99 = 3$\frac{3}{11}$ square rods.

### PROBLEM II.

EXAMPLES.

1. 4 × 4 + 3 = 19 roods, and 19 × 40 + 12 = 772 square rods, and 772 square rods divided by 16, because 16 square rods are equal to 1 square chain = 48,25 square chains in the area, and 9,20)48,25 = 5,2445 + chains, the length of the perpendicular.

2. 4 feet 8 inches reduced to a decimal becomes 4,666, and 47 feet 9 inches = 47,75; then 4,666)47,75 = 12,141 + feet, the length of the side.

3. 1 square rod = 272,25 feet; then 272,25 × 144 = 39204 square inches, and 7 × 12 + 2 = 86 inches, the length of the perpendicular; then 86)39204 = 37,99 — feet, the length.

## OF RIGHT-ANGLED TRIANGLES.

### PROBLEM I.

EXAMPLES.

1. 2)36 = 18 rods, half the length of the base; then 18 × 12 = 216 rods the area, = to 1 acre, 1 rood, and 16 rods.

2. 2)24,76 = 12,38, and 12,38 × 41,23 = 510,4274 square chains, and dividing by 10, the number of square chains in an acre, the area will be 51,04274 acres, and by finding the value of the decimal it will be found = to 6⅘ rods.

### PROBLEM II.

EXAMPLES.

1. 16,5 × 2 = 33 feet twice the area, and 33 feet × 144 = 4752, and 7 × 12 + 2 = 86; then 86)4752 = 55,256 — inches = to 4,604 + feet.

2. 4 × 4 + 1 = 17 roods, and 17 × 40 + 20 = 700 square rods the area, and 16)700 = 43,75 chains the area; then 43,75 × 2 = 87,50, and 3,14)87,50 = 27,8662 + chains, the length of the base.

### PROBLEM III.

EXAMPLES.

1. 16 × 16 = 256 the square of the base, and 12 × 12 = 144 the square of the perpendicular; then 256 + 144 = 400 the square of the hypotenuse; then $\sqrt{400}$ = 20 chains, the length of the hypotenuse.

2. 4,5 × 4,5 = 20,25 the square of the perpendicular, and 7,75 × 7,75 = 60,0625 square of the base; then 60,0625 × 20,25 = 80,3125 the square of the hypotenuse; then $\sqrt{80,3125}$ = 8,9617 + feet, the length of the hypotenuse.

### PROBLEM IV.

EXAMPLES.

1. 25 × 25 = 625 the square of the hypotenuse, and 20 × 20 = 400 the square of the base; then 625 — 400 = 225 the square of the perpendicular; then $\sqrt{225}$ = 15 chains, the perpendicular required.

2. 40 × 40 = 1600 the square of the hypotenuse, and 26 × 26 = 676 the square of the perpendicular; then 1600 — 676 = 924 the square of the base; then $\sqrt{924}$ = 30,38 + chains, the length of the base required.

3. 30 × 30 = 900 the square of the length of the ladder, and 18 × 18 = 324 square of the perpendicular on one side of the street; then 900 — 324 = 576 the square of the distance from one side of the street to the foot of the ladder; then $\sqrt{576}$ = 24 feet the distance from one side of the street to the foot of the ladder, and 24 × 24 = 576 the square of the perpendicular on the opposite side; then 900 — 576 = 324, and $\sqrt{324}$ = 18 the distance from the opposite side to the foot; then 18 + 24 = 42 feet, the width of the street.

4. 120 × 120 = 14400 the square of the distance from the place of observation to the top of the fort, and 80 × 80 = 6400 the square of the distance level with the eye; then 14400 — 6400 = 8000 the square of the height of the rock and fort together; then $\sqrt{8000}$ = 89,4427 yards, the height of the rock and fort; then 100 × 100 =

10000 the square of the distance from the place of observation to the top of the rock, and 6400 square of the distance level with the eye of the observer; then 10000 — 6400 = 3600 the square of the height of the rock; then $\sqrt{3600}$ = 60 yards, the height of the rock; then 89,4427 — 60 = 29,4427 + yards, the height of the fort.

PROBLEM V.

EXAMPLES.

1. 20 × 20 = 400 the square of the base, and 2 × 2 = 4 the square of the difference; then 400 — 4 = 396, and 2 × 2 = 4 twice the difference; then 4)396 = 99 chains, the perpendicular.

2. 24 × 24 = 576, and 5 × 5 = 25; then 576 — 25 = 551, and 5 × 2 = 10; then 10)551 = 55,1 chains, length of the perpendicular; then 2)24 = 12 chains, half the length of the base; then 55,1 × 12 = 661,2 the area in square chains; then 10)661,2 = 66,12 acres, the area required.

3. 24 × 24 = 576 the square of the base, and 6,14 × 6,14 = 37,6996; then 576 — 37,6996 = 538,3004, and 6,14 × 2 = 12,28 twice the difference; then 12,28)538004 = 43,8355 feet, the perpendicular required.

PROBLEM VI.

EXAMPLES.

1. 40 × 40 = 1600 the square of the base, and 4 × 4 = 16 the square of the difference; then 1600 + 16 = 1616, and 4 × 2 = 8 = twice the difference; then 8)1616 = 202 feet, the length of the hypotenuse.

2. 30 × 30 = 900 the square of the base, and 7 × 7 = 49 the square of the given difference; then 900 + 49 = 949, and 7 × 2 = 14 twice the given difference; then 14)949 = 67,8 — feet, the length of the hypotenuse required.

PROBLEM VII.

EXAMPLES.

1. 100 × 100 = 10000 the square of the sum, and 40 × 40 = 1600 the square of the base; then 10000 —

1600 = 8400, and 100 × 2 = 200 twice the sum, and 200)8400 = 42 feet, the length of the perpendicular required.

2. 120 × 120 = 14400 the square of the height of the tree, and 40 × 40 = 1600 the square of the distance from the base where the top may rest; then 14400 — 1600 = 12800, and 120 × 2 = 240 twice the length of the tree; then 240)12800 = 53 feet and 4 inches, the height of the upright part.

3. 50 × 50 = 2500, and 30 × 30 = 900; then 2500 — 900 = 1600, and 50 × 2 = 100; then 100)1600 = 16 chains the length of the perpendicular, and 50 — 16 = 34 chains, the hypotenuse.

### PROBLEM VIII.

#### EXAMPLES.

1. 60 × 60 = 3600 the square of the sum of the hypotenuse and perpendicular, and 24 × 24 = 576 the square of the base; then 3600 + 576 = 4176, and 60 × 2 = 120 twice the sum; then 120)4176 = 34,8 chains, the length of the hypotenuse.

2. 100 × 100 = 10000 the square of the length of the tree, and 30 × 30 = 900 the square of the base; then 10000 + 900 = 10900, and 100 × 2 = 200 twice the length of the tree; then 200)10900 = 54,5 feet, the length of the broken part.

## OF EQUILATERAL TRIANGLES.

### PROBLEM I.

#### EXAMPLES.

1. 20 × 20 = 400 the square of the side, and ,433013 the area of an equilateral triangle whose side is one; then ,433013 × 400 = 173,2052 square chains; then 10)173,2052 = 17,32052 acres.

2. 30 × 30 = 900 square feet; then 9)900 = 100 square yards, and ,433013 × 100 = 43,3013 square yards, the area.

3. $40 \times 40 = 1600$, and $,433013 \times 1600 = 692,8208$ square rods; then $160)692,8208 = 4,33005$ acres.

### PROBLEM II.

EXAMPLES.

1. $20 \times 20 = 400$ the square of the given side; then as the perpendicular in an equilateral triangle must fall on the middle of the side, therefore $2)20 = 10$ chains, the distance from the angles where the perpendicular strikes the base; then $10 \times 10 = 100$, and $400 - 100 = 300$ the square of the perpendicular; then $\sqrt{300} = 17,32 +$ chains, the perpendicular required.

2. $12 \times 12 = 144$, and $2)12 = 6$, and $6 \times 6 = 36$; then $144 - 36 = 108$ and $\sqrt{108} = 10,3923 +$.

### PROBLEM III.

EXAMPLES.

1. $11 \times 4 + 3 = 47$ roods, and $47 \times 40 + 16 = 1896$ the area in square rods; then $1896 \times 2 = 3792$, and $13,8564 \times 4 = 55,4256$ rods, the length of the perpendicular; then $55,4256)3792 = 68,416 +$ rods.

2. $1 \times 4 + 2 = 6$ roods, and $6 \times 40 + 15 = 255$ rods the area, and $255 \times 2 = 510$ square rods twice the area, $21,26$ rods the perpendicular; then $21,26)510 = 23,9887 +$ rods, the length of the side.

### PROBLEM IV.

EXAMPLE.

1. $11 \times 4 + 3 = 47$ roods, and $47 \times 40 + 16 = 1896$ square rods the area, and $1896 \times 2 = 3792$ square rods in twice the area, and $16)3792 = 237$ square chains in twice the area, and $16)237 = 14,8125$ chains, the perpendicular required.

### PROBLEM V.

EXAMPLES.

1. $24 \times 10 = 240$ square chains the area, and $,433013)240 = 554,2558$ the square of the side; then

$\sqrt{554,2558} = 23,5426$ chains, the length of the side required.

2. 4 acres, 2 roods, and 5 rods = 725 square rods in the area; then 16)725 = 45,3125 square chains the area; then ,433013)45,3125 = 104,6446 the square of the side; then $\sqrt{104,6446} = 10,229 +$ chains, the length of the side.

3. 1 chain = 66 feet; then $66 \times 66 = 4356$ square feet the area, and ,433013)4356 = 10059,744 the square of the side; then $\sqrt{10059,744} = 100,29$ feet, the side of the triangle.

#### PROBLEM VI.

##### EXAMPLES.

1. $\sqrt{3} = 1,732052$, and 1,73205 = ,866026; then ,866026)45,40 = 52,4233 chains.

2. $26 + 32 + 19 = 77$ rods the sum of the three perpendiculars, and ,866026 half the square root of the number 3; then ,866026)77 = 88,9 rods, the length of the side; then $88,9 \times 88,9 \times ,433013 = 3422,19 +$ and 160)3422,19 + = 21,39 — acres.

3. $16,5 + 24 + 32,5 = 73$ feet the sum of the 3 perpendiculars; then ,866026)73 = 84,29 feet the length of the side of the triangle; then $84,29 \times 84,29 \times ,433013 = 307,6472 +$ square feet the area; then 272,25 square feet in 1 square rod; then 272,25)307,6472 + = 1,13 + square rods, the area required.

## OF THE ISOSCELES TRIANGLE.

#### PROBLEM I.

##### EXAMPLES.

1. $24 \times 24 = 576$ the square of one of the equal sides, and 2)32 = 16 feet; then $16 \times 16 = 256$ the square of half the base; then $576 - 256 = 320$ the square of the perpendicular; then $\sqrt{320} = 17,8885 +$ the length of the perpendicular; then 2)32 = 16 feet half the base of the triangle, and $17,8885 \times 16 = 286,216$ the number of square feet the triangle contains,

and 272,25 the number of square feet in one square rod; therefore 272,25)286,216 = 1,0513 — rods, the area of the triangle.

2. 30 × 30 = 900 the square of one of the equal sides, and 2)25 = 12,5 chains half the base, and 12,5 × 12,5 = 156,25 the square of half the base, and 900 — 156,25 = 743,75 the square of the perpendicular, and $\sqrt{743,75}$ = 27,2717 + chains the length of the perpendicular; then 27,2717 × 12,5 = 340,89625 square chains = to 34 acres and 14 rods.

#### PROBLEM II.

##### EXAMPLES.

1. 4 × 2 = 8 square rods twice the area; then 272,-25 × 8 = 2178 square feet in twice the area, and 18)2178 = 121 feet the length of the perpendicular, and 121 × 121 = 14641 the square of the perpendicular, and 2)18 = 9 feet half the base; then 9 × 9 = 81 the square of half the base; then 14641 + 81 = 14722 the square of one of the equal sides; then $\sqrt{14722}$ = 121,3342 feet, the length of each of the equal sides.

2. 48 × 2 = 96 square rods twice the area; then 8)96 = 12 rods the perpendicular; then 12 × 12 = 144 the square of the perpendicular, and 2)8 = 4 rods half the base; then 4 × 4 = 16 the square of half the base; then 144 + 16 = 160 the square of the equal side; then $\sqrt{160}$ = 12,649 + rods, the length of each of the equal sides.

### OF SCALENE TRIANGLES.

#### PROBLEM I.

##### EXAMPLES.

1. 2)18 = 9 rods half the length of the base, and 9 × 9 = 81 square rods the area; then 16)81 = 5,1 square chains, the answer required.

2. 15 × 66 = 990 feet the length of the base; then 2)18 = 9 feet half the perpendicular; then 990 × 9 = 8910 feet the area; then 8910 ÷ 272,25 = 32,7272 sq. rods, the area.

3. $160 \times 4 = 640 \times 2 = 1280$, twice the area in rods, and $18)1280 = 71\frac{1}{9}$ rods, the length of the perpendicular.

### PROBLEM II.

#### EXAMPLES.

1. $16 + 18 + 24 = 58$ chains, the sum of the 3 sides; then $2)58 = 29$ the half sum; then $29 - 16 = 13$, and $29 - 18 = 11$, and $29 - 24 = 5$; then $29 \times 13 \times 11 \times 5 = 20735$; then $\sqrt{20735} = 144$ — square chains; then $10)144 = 14,4$ — acres nearly.

2. $12 + 16 + 20 = 48$, and $2)48 = 24$ rods the half sum of the 3 sides; then $24 - 12 = 12$, and $24 - 16 = 8$, and $24 - 20 = 4$; then $24 \times 12 \times 8 \times 4 = 9216$, and $\sqrt{9216} = 96$ square rods, the area required.

3. $24 + 36 + 44 = 104$ feet the sum of the 3 sides; $2)104 = 52$ the half sum; then $52 - 24 = 28$, and $58 - 36 = 22$, and $58 - 44 = 14$; then $52 \times 28 \times 22 \times 14 = 448448$ the square of the area; then $\sqrt{448448} = 669,6626$ square feet; then $272,25)669,6626 = 2,46$ — square rods nearly.

### PROBLEM III.

#### EXAMPLES.

1. $20 : 14 + 18 :: 18 - 14 : 6,4$ the difference of the segments of the base; then $2)6,4 = 3,2$ feet half their difference; then $2)20 = 10$ feet half the base; then $10 + 3,2 = 13,2$ the longer of the two segments, and $10 - 3,2 = 6,8$ feet the shorter, and 14 feet the shorter side; then $14 \times 14 = 196$ the square of the shorter side, and $6,8 \times 6,8 = 46,24$ the square of the shorter segment; then $196 - 46,24 = 149,76$, and $\sqrt{149,76} = 12,2376 +$ feet, the length of the perpendicular.

2. $80 : 60 + 40 :: 60 - 40 : 25$ the difference of the two segments; then $2)25 = 12,5$ chains half the difference, and $2)80 = 40$ chains half the base; then $40 + 12,5 = 52,5$ chains the greater of the two segments,

and $40 - 12,5 = 27,5$ chains the less of the segments; then $40 \times 40 = 1600$ the square of the shorter side, and $27,5 \times 27,5 = 756,25$ the square of the less segment; then $1600 - 756,25 = 843,75$ chains the square of the perpendicular; then $\sqrt{843,75} = 29,0473 +$ chains, the length of the perpendicular.

### PROBLEM IV.

#### EXAMPLES.

1. $3 + 4 + 6 = 13$ chains the sum of the sides given, and $2)13 = 6,5$ the half sum; then $6,5 - 3 = 3,5$, and $6,5 - 4 = 2,5$, and $6,5 - 6 = ,5$; then $6,5 \times 3,5 \times 2,5 \times ,5 = 28,4375$ the square of the area; then $\sqrt{28,4375} = 5,3326 +$ the area of the triangle given; then $5,3326 : 3 \times 3 :: 10$ chains : $16,8773$ the square of the shorter side of the required triangle; then $\sqrt{16,8773} = 4,1082 -$ chains the shortest side; then $3 : 4,1082 :: 4 : 5,4776$ chains the second side, and $3 : 41082 :: 6 : 8,2164$ chains, the longest side.

2. $120 \times 4 + 3 = 483$ roods, and $483 \times 40 + 4 = 19324$ square rods the area of the triangle whose sides are required, and $9 + 8 + 6 = 23$, sum of the proportionated sides; then $2)23 = 11,5$ the half sum, and $11,5 - 9 = 2,5$, and $11,5 - 8 = 3,5$, and $11,5 - 6 = 5,5$; then $11,5 \times 2,5 \times 3,5 \times 5,5 = 553,4375$ the square of the area according to the given proportion; then $\sqrt{553,4375} = 23,5252 +$ square rods the area; then $23,5252 : 6 \times 6 :: 19324 : 29571$ rods the square of the shortest side; then $\sqrt{29571} = 172 -$ rods the length of the shortest side; then $6 : 172 :: 9 : 258$ the length of the longest side, and $6 : 172 :: 8 : 229\frac{1}{3}$ the other.

3. $5 + 11 + 13 = 29$ the sum of the sides in the given proportion; then $2)29 = 14,5$ chains the half sum, and $14,5 - 5 = 9,5$, and $14,5 - 11 = 3,5$, and $14,5 - 13 = 1,5$; then $14,5 \times 9,5 \times 3,5 \times 1,5 = 723,1875$ the square of the area corresponding with the given proportion; then $\sqrt{723,1875} = 26,892$ chains the area; then $26,892 : 5 \times 5 :: 100 : 92,9644$ the square

of the shortest side; then $\sqrt{92,9644} = 9,64$ chains the shortest side; then $5 : 9,64 :: 11 : 21,208$ the second side, and $5 : 9,64 :: 13 : 25,064$ chains, the longest side.

#### PROBLEM V.

##### EXAMPLES.

1. 5 acres, 2 roods, 25 rods $= 905$ rods the area, and 40 chains $\times 4 = 160$ rods, and $2)160 = 80$ rods the half sum of the 3 sides; then $80 \times 80 = 6400$ the square of the half sum, and $18 \times 4 = 72$ rods the length of the given side; then $160 - 72 = 88$ rods the sum of the two unknown sides; then $88 - 80 = 8$ rods the difference, and $8 \times 8 = 64$ the square of the difference; then $6400 \times 64 = 409600$, and $905 \times 905 = 819025$ the square of the area, and $409600 + 819025 = 1228635$ the dividend, and $88 - 80 = 8$; then $80 \times 8 = 640$ divisor, and $640)1228635 = 1919,74219$ —, and $2)88 = 44$, and $44 \times 44 = 1936$; then $1936 - 1919,74219 = 16,25781$, and $\sqrt{16,25781} = 4,032$ half the difference of the two unknown sides; then $2)88 = 44$ the half sum, and $44 + 4,032 = 48,032$ rods the longer of the two sides, and $44 - 4,032 = 39,968$ rods, the shorter side.

2. $50 + 20 = 70$ rods the sum of the three sides, and $2)70 = 35$ rods the half sum; then $35 \times 35 = 1225$ the square of the half sum, and $50 - 35 = 15$ rods the difference; then $15 \times 15 = 225$ the square of the difference; then $1225 \times 225 = 275625$, and $160 \times 160 = 25600$ the square of the area; then $275625 + 25600 = 301225$ the dividend, and $35 \times 15 = 525$ the divisor; then $525)301225 = 573,7619$, and $2)50 = 25$ the half sum of the two unknown sides; then $25 \times 25 = 625$ the square, and $625 - 573,7619 = 51,2381$, and $\sqrt{51,2381} = 7,158$ rods half the difference of the two unknown sides, and 25 the half sum; then $25 + 7,158 = 32,158$ rods the longer side, and $25 - 7,158 = 17,842$ rods, the shorter side.

3. $20 + 30 = 50$ feet the sum of the three sides; then $2)50 = 25$ the half sum, and $25 \times 25 = 625$ the

square, and 30 — 25 = 5 the difference, and 5 × 5 = 25 the square of their difference; then 625 × 25 = 15625, and 11375 the square of the area given in the question; then 15625 + 11375 = 27000 the dividend, and 25 × 5 = 125 the divisor; then 125)27000 = 216 the product of the two required sides; then 2)30 = 15 half the sum of the required sides, and 15 × 15 = 225, and 225 — 216 = 9 the square of half their difference; then $\sqrt{9}$ = 3 half their difference, and 15 + 3 = 18 feet the longer side, and 15 — 3 = 12 feet, the shorter.

## OF THE TRAPEZIUM.

### PROBLEM I.

#### EXAMPLES.

1. 24,27 × 21,14 = 513,0678, and 2)513,0678 = 256,5339 chains the area; then 10)256,5339 = 25, 65339 acres = 25 acres, 2 roods, and 24 rods.

2. 33 × 24 = 792, and 2)792 = 396 rods; then 160)396 = 2 acres, 1 rood, and 36 rods.

3. 56 feet 3 inches + 60 feet 9 inches = 117 feet the sum of the perpendiculars; 108,5 × 117 = 12694,5, and 2)12694,5 = 6347,25 square feet the area, and 272,25)6347,25 = 23,314 square rods, the answer required.

4. 22,4 + 28,3 = 50,7 chains the sum of the perpendiculars, and 50,7 × 80,5 = 4081,35, and 2)4081,35 = 2040,675 square chains the area; then 10)2040,675 = 204,0675 acres = 204 acres, 2 roods, and 28 rods.

## OF THE TRAPEZOID.

### PROBLEM I.

#### EXAMPLES.

1. 24,46 + 38,40 = 62,86 chains the sum of the two parallel sides; then 62,86 × 16,2 = 1018,332 the product of the sum of the parallel sides by the perpendicular; then 2)1018,332 = 509,166 square chains; then 10)509,166 = 50,9166 acres = 50 acres, 3 roods, and 26 rods.

2. 12,75 + 16,67 = 29,42 rods the sum of the two parallel sides, and 29,42 × 4,5 = 132,39, and 2)132,39 = 66,195 square rods, the area required.

3. 14,5 + 24,75 = 39,25 feet the sum of the two parallel sides; then 39,25 × 8,25 = 323,8125, and 2)323,8125 = 161,9 + square feet, the area.

4. 5,16 + 9,14 = 14,30 chains the sum of the two parallel sides, and 14,3 × 3,07 = 43,9, and 2)43,9 = 21,95 square chains; then 21,95 × 16 = 351,2 square rods.

## OF REGULAR POLYGONS.

### PROBLEM I.

#### EXAMPLES.

1. 25 × 5 = 125 feet the perimeter; then 2)125 = 62,5 feet half the perimeter; then 62,5 × 17,2 = 1075 feet, the area required.

2. 14,6 × 6 = 87,6 feet the perimeter; then 2)87,6 = 43,8 half the perimeter; then 43,8 × 12,64 = 553,-632 square feet the area, and 272,25)553,632 = 2,0335 square rods, the area required.

3. 19,38 × 7 = 135,66 chains the perimeter; then 2)135,66 = 67,83 chains half the perimeter, and 67,83 × 20 = 1356,6 square chains the area; then by dividing by 10 = 135,66 acres, the answer required.

4. 9,941 × 8 = 79,528 feet the perimeter, and 2)79,-528 = 39,764 feet half the perimeter; then 39,764 × 12 = 477,168 square feet the area; then 272,25)477,-168 = 1,7526 square rods the area required.

### PROBLEM II.

#### EXAMPLES.

1. 25 × 25 × 1,720477 = 1075,298 + feet, the area.

2. 24 × 24 × 2,598076 = 1496,49 + square feet.

3. 16 × 16 × 3,633912 = 930,28 + square feet, the area required.

4. 12,5 × 12,5 × 4,828427 = 754,4417 + square feet; then 9)754,4417 = 83,8268 + square yards, the area required.

5. $15 \times 15 \times 6{,}181824 = 1390{,}91 +$ square chains $=$ to 139 acres and 14 perches.

6. $12 \times 12 \times 7{,}694209 = 1107{,}966$ square inches; then $4 \times 4 = 16$ square inches in one piece, and $16)1107{,}966 = 69{,}248 -$ pieces.

7. $4 \times 4 \times 9{,}36564 = 149{,}85 +$ square rods, the area.

8. $9 \times 9 \times 11{,}196152 = 906{,}888 +$ square inches; then $144)906{,}888 = 62{,}98 -$ square feet nearly.

#### PROBLEM III.

##### EXAMPLES.

1. $4 \times 160 = 640$ perches; then $1{,}720477)640 = 371{,}9898$ the square of the side; then $\sqrt{371{,}9898} = 19{,}287 +$ perches, the length of the side.

2. $4{,}828427)560 = 115{,}9798$ the square of the side; then $\sqrt{115{,}9798} = 10{,}769 +$ feet, the length of the side.

3. $2{,}598076)160 = 61{,}584$ the square of the side; then $\sqrt{61{,}584} = 7{,}8475 +$ perches, the length of the side.

### TO DIVIDE A SQUARE.

#### PROBLEM I.

##### EXAMPLES.

1. $15 \times 10 = 150$ square chains the area, and $16)150 = 9{,}375$ chains, the answer required.

2. $10 \times 10 = 100$ square chains the area; then $20{,}14)100 = 4{,}985$ chains.

3. $5 \times 4 + 2 = 22$, and $22 \times 40 + 14 = 894$ square rods the area, and $\sqrt{894} = 29{,}9 -$ rods nearly, length of the side; then $3 \times 4 = 12$ roods, and $12 \times 40 + 6 = 486$ square rods the area to be cut off; then $29{,}9)486 = 16{,}254 +$ rods, the length of the required side.

#### PROBLEM II.

##### EXAMPLES.

1 $2)10{,}18 = 5{,}09 \times 10{,}18 = 51{,}8162$ square chains half the area of the square; then $51{,}3162 : 10{,}18 \times 10{,}18 :: 40$ chains the area left : 80 chains the square

of the required side; then $\surd 80 = 8{,}9442 +$ chains, the sides required.

2. $2)160 = 80$ square rods half the area, and 160 rods the area, and consequently the square of one of its sides; then $80 : 160 :: 25 : 50$ rods the square of the side of the isosceles triangle required; then $\surd 50 = 7{,}071$ rods, the length of the side.

## OF THE DIVISION OF A PARALLELOGRAM.

### PROBLEM I.

#### EXAMPLES.

1. $18{,}16 \times 12{,}15 = 220{,}644$ square chains the area; then $220{,}644 - 120 = 100{,}644$; then $12{,}15)100{,}644 = 8{,}2834 +$ chains, the length of the side required.

2. $4 \times 10 = 40$ square chains; then $12)40 = 3\frac{1}{3}$ chains.

### PROBLEM II.

#### EXAMPLES.

1. $12{,}48 \times 18 = 224{,}64$, and $2)224{,}64 = 112{,}32$ chains half the area of the triangle, and $18 \times 18 = 324$ the square of the longest side; then $112{,}32 : 324 :: 70 : 201{,}9237$ the square of the longer side of the triangle; then $\surd 201{,}9237 = 14{,}21 -$ chains the longer side, and $12{,}48 \times 12{,}48 = 155{,}7504$ the square of the shorter side; then $112{,}32 : 155{,}7504 :: 70 : 97{,}0666$ the square of the shortest side of the required triangle; then $\surd 97{,}0666 = 9{,}85 +$ chains, the shorter side.

2. $5 \times 4 = 20$ rods the shortest side, and $12 \times 4 + 3 = 51$ roods, and $51 \times 40 = 2040$ square rods the area of the parallelogram; then $2)2040 = 1020$ square rods half the area; then $1020 : 20 \times 20 :: 720$ rods $= 4{,}5$ acres : $282{,}353 -$ the square of the shortest side of the triangle; then $\surd 282{,}353 = 16{,}8 +$ rods the shortest side, and $720 \times 2 = 1440$ square rods twice the area of the triangle; then $16{,}8)1440 = 85{,}7 +$ rods, the length of the longest side.

3. $9 \times 4 + 2 = 38$ roods, and $38 \times 40 + 16 = 1536$ square rods the area; then $16)1536 = 96$ square chains

the area; then $96 \times 2 = 192$, and $3)192 = 64$ chains the square of the shortest side of the parallelogram; then $\sqrt{64} = 8$ chains the shortest side; then $2 : 8 :: 3 : 12$ chains the longest side; then $96 : 12 \times 12 :: 30$ chains : 45, the square of the longest side of the required triangle; then $\sqrt{45} = 6,708 +$ the longest of the proportional sides; then $3 : 6,708 :: 2 : 4,472$ chains the shortest side.

## OF THE DIVISION OF TRIANGLES.

### PROBLEM I.

#### EXAMPLES.

1. $2 \div 20 = 10$ chains half the base; then $12,14 \times 10 = 121,4$ the area of the triangle, and $2 \div 121,4 = 60,7$ square chains half the area, and $20 \times 20 = 400$ chains the square of the base; then $121,4 : 400 :: 60,7 : 200$ the square of the required base; then $\sqrt{200} = 14,1421 +$ chains the length of the base, measuring from the vertical angle, and $20 - 14,1421 = 5,8579 -$ the other part.

2. $2 \div 12 = 6$ rods half the perpendicular; then $18 \times 6 = 108$ rods the area of the triangle; then $108 \times 2 = 216$, and $3 \div 216 = 72$ square rods $= \frac{2}{3}$ of the area, the quantity left at the vertical angle; then $108 : 12 \times 12 :: 72 : 96$, and $\sqrt{96} = 9,798 -$; then $12 - 9,798 = 2,002 +$ rods the length of the perpendicular of the section adjoining the base; then $3 \div 108 = 36\frac{1}{3}$ of the whole area; then $108 : 12 \times 12 :: 36 : 48$ the square of the perpendicular of the section left at the vertical angle; then $\sqrt{48} = 6,921 -$ rods, length of the perpendicular from the vertical angle; then $9,798 - 6,921 - = 2,877$ rods length of the perpendicular of the middle section; therefore the lengths $6,921 -$, $2,877$, and $2,202$ rods required.

3. $2 + 3 + 4 = 9$ the sum of the proportional numbers; then $9 - 4 = 5$ the proportional quantity left at the vertical angle after the greater proportional part has been cut off, and $24 \times 24 = 576$ the square of the base; then $9 : 576 :: 5 : 320$ the square of the remaining part

of the base after the greater section has been cut off; then $\sqrt{320} = 17{,}889 +$ chains the length of the base after the first division; then $24 - 17{,}889 + = 6{,}111 -$ chains length of the base of the greater section, and $9 - 7 = 2$ the proportional quantity at the vertical angle; then $9 : 576 :: 2 : 128$ the square of the base of the less part; then $\sqrt{128} = 11{,}3137 +$ chains length of the base of the section left at the vertical angle; then $17{,}889 - 11{,}3137 = 6{,}5753$ chains length of the base of the middle section; then the lengths are $11{,}3137 -$, $6{,}5753$, and $6{,}111$ chains, the answer required.

### PROBLEM II.

#### EXAMPLES.

1. $12 \times 12 \times ,433013 = 62{,}353872$ chains the area of the given triangle; then $62{,}353872 \times 2 \div 3 = 41{,}569248$ the area left at the vertical angle after the section adjoining has been cut off; then $62{,}353872 : 12 \times 12 :: 41{,}569248 : 96$ the square of the side left after the first division; then $\sqrt{96} = 9{,}798 -$ the length of the side where the second division line must be drawn, and $3 \div 62{,}353872 = 20{,}784624$ area left after the second division; then $6{,}2353872 : 12 \times 12 :: 20{,}784624 : 48$ the square of the side left after the second division; then $\sqrt{48} = 6{,}921 -$ chains the side left after the second division; therefore the distances from the vertical angle are $6{,}921$ and $9{,}798$ chains.

2. $12 + 18 + 20 = 50$ rods the sum of the three sides, and $2 \div 50 = 25$ the half sum, and $25 - 20 = 5$, and $25 - 18 = 7$, and $25 - 12 = 13$; then the differences between the half sum and each of the given sides are 5, 7, and 13; then $25 \times 5 \times 7 \times 13 = 11375$ the square of the area; therefore $\sqrt{11375} = 106{,}6532 -$ square rods the area of the given triangle; then $2 \div 106{,}6532 - = 53{,}3266 -$ half the area; then $106{,}6532 : 12 \times 12 :: 53{,}3266 : 72$ the square of the distance required on the shortest side; then $\sqrt{72} = 8{,}485$ rods from the angle opposite the base where the division line must be drawn on the shortest side. To find

the distance on the other side, proceed in the same manner with the square of 18 the given side.

3. $4+5+7=16$ the sum of the three sides; then $2 \div 16 = 8$ the half sum; then $8-4=4$, and $8-5=3$, and $8-7=1$; then the three differences are 4, 3, and 1; then $8 \times 4 \times 3 \times 1 = 96$ the square of the area according to the given proportion; then $\sqrt{96} = 9,798$ — square chains, the area of a triangle whose sides are 4, 5, and 7 chains; then $9,798 : 7 \times 7 :: 21 \times 10 : 1050,21$ the square of the base of a triangle of similar proportions containing 21 acres; then $\sqrt{1050,21} = 32,4$ chains the length of the base; then $21 \times 10 \times 2 = 420$ chains twice the area, and $32,4 \div 420 = 13$ — chains the length of the perpendicular of the triangle containing 21 acres, and $2+3+5=10$ the sum of the proportional parts of the sections, and $10-5=5$ the proportional part left at the angle opposite the base; then $10 : 13 \times 13 :: 5 : 84,5$ the square of the distance from the vertical angle to the line cutting off the proportional part adjoining the base; then $\sqrt{84,5} = 9,0194$ + chains the distance, and $10-8=2$ the proportional quantity left at the vertical angle after the second division; then $10 : 13 \times 13 :: 2 : 33,8$ the square of the distance from the vertical angle to the second division line; then $\sqrt{33,8} = 5,0872$ chains the distance; then $13 - 9,0194 = 3,9806$ chains the perpendicular distance between the two greater proportions, and $9,0194 - 5,0872 = 3,9322$ chains, the perpendicular distance between the two less.

4. $3 \times 2 = 6$, and $2 \div 6 = 3$ square chains the area of the proportional triangle, and $12 \times 10 = 120$ square chains the area given in the question; then $3 : 2 \times 2 :: 120 : 160$ the square of the perpendicular of the triangle containing 12 acres; then $\sqrt{160} = 12,649$ + chains the length of the perpendicular, and $3 : 3 \times 3 :: 120 : 360$ the square of the base, and $\sqrt{360} = 18,9736$ + chains the length of the base; then $12 \times 10 = 120$ chains the area, and $3)120 = 40$ chains the one-third part of the area to be cut off; then $120 - 40 = 80$

square chains to be left at the vertical angle; then 120 : 160 the square of the perpendicular :: 80 : 106,6666 the square of the perpendicular left after the first division; then $\sqrt{106,6666} = 10,328$ — the length of the perpendicular, and 120 : 160 :: 40 : 53,3333 the square of the perpendicular left after the second division; then $\sqrt{53,3333} = 7,303$ — the length of the perpendicular after the second division; then $12,649 - 10,328 = 2,321$, and $10,328 - 7,303 = 3,025$ chains, the distances required.

# SECTION II.

## MENSURATION OF CIRCLES.

### PROBLEM I.

EXAMPLES.

1. 3,1416 × 12 = 37,6992 rods, the circumference required.

2. 3,1416 × 18,3 = 57,4918 chains.

3. 5 × 12 + 2 = 62 inches the diameter; then 3,1416 × 62 = 194,7792 inches = 16 2316 feet.

### PROBLEM II.

EXAMPLES.

1. 3,1416)16 = 5,0929 + chains, the diameter required.

2. 16,5 × 2 = 33 feet; 3 ÷ 33 = 11 feet the circumference of the wheel; 3,1416)11 = 3,5 + feet, the diameter required.

3. 16,5 ÷ 3,1416 = 5,2521 + feet, the diameter.

### PROBLEM III.

EXAMPLES.

1. 16 × 16 × ,7854 = 201,0624 square chains, which divided by 10 = 20,10624 = 20 acres, 0 roods, and 17 perches nearly.

2. 18 × 18 × ,7854 = 254,4696 square rods the area, and 160)254,4696 = 1,59 + acres, the area required.

3. 4,25 × 4,25 × ,7854 = 14,1862875 square feet.

4. 16,5 × 6 = 99 feet the diameter; then 99 × 99 × ,7854 × ,08 = 615,816432 dollars.

### PROBLEM IV.

EXAMPLES.

1. 5 × 4 + 3 × 40 + 26 = 946 square rods the area of the circle; then ,7854)946 = 1204,48 rods the square

of the diameter; then $\sqrt{1204,48} = 34,7$ rods, the diameter.

2. ,7854)5 = 6,3662 — chains the square of the diameter; then $\sqrt{6,3662} = 2,523 +$ chains, the diameter required.

3. $10 \times 2 = 20$ chains the area, and ,7854)20 = 25,4647 + chains the square of the diameter of a circle containing two acres; then $\sqrt{25,4647} = 5,0462 +$ the diameter; then $2 \div 5,0462 = 2,5231$ chains, the length of the rope.

4. $160 + 120 = 280$ rods the area given; then ,7854)280 = 356,5 rods the square of the diameter; then $\sqrt{356,5} = 18,8812 +$ rods the diameter, and $18,8812 \times 16,5 \times 12 = 3738,4776$ inches the diameter; then $3738,4776 \times 2 \div 3 = 2492,3184$ dollars, the answer.

### PROBLEM V.

#### EXAMPLES.

1. $24 \times 24 \times ,07958 = 45,83808$ square chains, which divided by 10 gives 4,583808 = to 4 acres, 2 roods, and 13 perches.

2. $120 \times 120 \times 07958 = 1145,95$ 2square rods the area; then 160)1145,952 = 7,1622 acres, the answer required.

3. $5,5 \times 5,5 \times 07958 \times 9 = 21,665 +$ square feet, the area required.

### PROBLEM VI.

#### EXAMPLES.

1. $2 \times 4 + 3 = 11$ roods, and $11 \times 40 + 12 = 452$ square rods the area; then ,07958)452 = 5680 — the square of the circumference; then $\sqrt{5680} = 75,3657$ rods, the circumference required.

2. ,07958)160 = 2010,5554 rods the square of the circumference; then $\sqrt{2010,5554} = 44,839 +$ rods, the circumference required.

3. 4)46,50 = 11,625 square rods the area; then $,07958 \div 11,625 = 146,08$ — rods the square of the

circumference; then $\sqrt{146,08} = 12,0863 +$ rods the circumference; $16 \times 12 + 6 = 198$ the number of inches in one rod; then $198 \times 2 = 396$, and $3)396 = 132$ the number of dollars which would enclose one rod; then $12,0863 \times 132 = 1595,3916$ dollars, the answer required.

### PROBLEM VII.

#### EXAMPLES.

1. $3,1416 \times 100 = 314,16$ feet the circumference; then $360 : 314,16 :: 16 : 13,9626$ feet, the length of the arc required.

2. $3,1416 \times 36,25 = 113,883$ feet the circumference; then $360 : 113,883 :: 30 : 9,49 +$ feet, the length of the arc required.

3. $360 : 24,64 :: 41 : 2,8062$ chains, the length of the arc required.

### PROBLEM VIII.

#### EXAMPLES.

1. $2)12 = 6$ feet half the chord; then $6 \times 6 = 36$ the square of half the chord; then $2)36 = 18$, and $18 + 2 = 20$ feet, the diameter required.

2. $2)2,16 = 1,08$ the half chord; then $1,08 \times 1,08 = 1,1664$ the square of half the chord; then $,80)11664 = 1,458$, and $1,458 + ,80 = 2,258$ chains, the diameter required.

3. $2)5 = 2,5$ rods the half chord; then $2,5 \times 2,5 = 6,25$ the square of half the chord; then $2)625 = 3,125 + 2 = 5,125$ rods, the diameter required.

### PROBLEM IX.

#### EXAMPLES.

1. $4 \times 12 + 6 = 54$ inches the diameter of the circle; then $54 - 7 = 47$, and $47 \times 7 = 329$ inches the square of half the chord; then $\sqrt{329} = 18,1383$ inches half the chord; then $18,1383 \times 2 = 36,2766$ inches the chord, $= 3,023$ feet.

2. $18 - 2 = 16 \times 2 = 32$ feet the square of half the

chord; then $\sqrt{32} = 5,6568 +$ feet half the chord; then $5,6568 \times 2 = 11,3136$ feet, the length of the chord.

3. $30 - 3 = 27 \times 3 = 81$ chains the square of half the chord; then $\sqrt{81} = 9$ chains the half chord, and $9 \times 2 = 18$ chains, the length of the chord.

### PROBLEM X.

#### EXAMPLES.

1. $2)48 = 24$ feet half the chord, and $24 \times 24 = 576$ the square of half the chord, and $18 \times 18 = 324$ the square of the versed sine; then $576 + 324 = 900$ the sum of their squares, and $\sqrt{900} = 30$ feet the length of the diagonal; $30 - 24 = 6$ feet the difference between the diagonal and half the chord; then $6 \times 5 = 30$, and $14)30 = 2,143 -$; then $30 + 2,143 = 32,143 -$ feet, the length of half the arc; then $32,143 \times 2 = 64,286$ feet, the length of the arc.

2. $2)40 = 20$; then $20 \times 20 = 400$ the square of half the chord, and $15 \times 15 = 225$ the square of the versed sine, and $400 + 225 = 625$ the sum of their squares; then $\sqrt{625} = 25$ the diagonal; $25 - 20 = 5$ the difference between the diagonal and half the chord; then $5 \times 5 = 25$, and 25 divided by $14 = 1,7857 + 25 = 26,7857$ half the length of the arc; then $26,7857 \times 2 = 53,5714$ length of the arc.

3. $36 - 2 = 34$, and $34 \times 2 = 68$ the square of half the chord; then $2 \times 2 = 4$ the square of the versed sine; then $68 + 2 = 70$ the sum of their squares, and $\sqrt{70} = 8,3666 +$ the length of the diagonal, and $\sqrt{68} = 8,2462$; then $8,3666 - 8,2462 = ,1204$; then $1204 \times 5 = ,6020$ which divided by $14 = ,043$; then $8,3666 + 043 = 8,4096$ half the arc; then $8,4096 \times 2 = 16,8192$ the length of the arc required.

4. $60 - 12 = 48$, and $48 \times 12 = 576$ the square of half the chord; then $\sqrt{576} = 24$ the half chord, and $12 \times 12 = 144$ the square of the versed sine; then $576 + 144 = 720$ the sum of their squares, and $\sqrt{720} = 26,$-$8328 +$ the length of the diagonal; then $26,8328 - 24$

= 2,8328; 2,8328 $\times$ 5 = 14,164; then 14)14,164 = 1,0117 +; then 26,8328 + 1,0117 = 27,8445 half the arc, and 27,8445 $\times$ 2 = 55,689 the length of the arc required.

### PROBLEM XI.

#### EXAMPLES.

1. 2)40 = 20 half the chord; then 20 $\times$ 20 = 400, and 15)400 = $26\frac{2}{3}$, and $26\frac{2}{3}$ + 15 = $41\frac{2}{3}$, the diameter of the circle, and 15 $\times$ 15 = 225 the square of the versed sine, and 400 the square of half the chord; then 400 + 225 = 625 the sum of their squares, and $\sqrt{625}$ = 25 the length of the diagonal; then 25 — 20 = 5 the difference between the diagonal and half the chord; then 5 $\times$ 5 = 25, and 25 divided by 14 = 1,7857 +, and 25 + 1,7857 = 26,7857 + half the arc, and 2)$41\frac{2}{3}$ = $20\frac{5}{6}$ half the diameter = 125 divided by 6; then 26,7857 $\times$ 125 and divided by 6 = 558,0354 the area of the sector required.

2. 50 — 12 = 38, and 38 $\times$ 12 = 456 the square of half the chord; then 12 $\times$ 12 = 144 the square of the versed sine, and 456 + 144 = 600 the sum of their squares, and $\sqrt{600}$ = 24,495, and $\sqrt{456}$ = 21,354 + the half chord, and 24,495 — 21,354 = 3,141, and 3,141 $\times$ 5, and divided by 14 + 24,495 = 25,616 inches half the length of the arc, and 2)50 = 25 inches half the diameter; then 25,616 $\times$ 25 = 640,4 square inches, the area required.

3. 30 $\times$ 30 = 900 inches the square of half the chord, and 18 $\times$ 18 = 324 the square of the versed sine; then 900 + 324 = 1224 the sum of their squares, and $\sqrt{1224}$ = 35 — inches the diagonal; then 35 — 30 = 5 inches the difference, and 5 $\times$ 5, and divided by 14 = 1,7857, and 35 + 1,7857 = 36,7857 inches half the arc, and 30 $\times$ 30 = 900 the square of half the chord, and 18)900 = 50, and 50 + 18 = 68 inches the diameter of the circle, and 2)68 = 34 half the diameter, and 36,7857 $\times$ 34 = 1250,7138 inches, the area required.

4. 36 $\times$ 36 $\times$ ,7854 = 1017,8784 feet the area of the

circle; then 360 : 1017,8784 :: 24 : 67,8585 + feet, the area of the sector.

5. 28 × 28 × ,7854 = 615,7536 square chains the area of the circle; then 360 : 615,7536 :: 124 : 212,-0929 + square chains, the area of the sector.

### PROBLEM XII.

#### EXAMPLES TO RULE I.

1. 2)20 = 10 chains half the chord; then 10 × 10 = 100 the square of half the chord, and 4 × 4 = 16 the square of the versed sine; then 100 + 16 = 116 the sum of their squares, and √116 = 10,77 + chains the length of the diagonal, and 10,77 — 10 = ,77, and ,77 × 5 ÷ 14 = ,275, and 10,77 + ,275 = 11,045 chains the length of half the arc, and 4)100 = 25, and 25 + 4 = 29 chains the diameter of the circle; then 2)29 = 14,5 half the diameter; then 14,5 × 11,045 = 160,1525 the area of the sector; then 14,5 — 4 = 10,5 chains the perpendicular of the triangular part of the sector; then 10,5 × 10 = 105 square chains the area of the triangular part of the sector, and 160,1525 — 105 = 55,1525 square chains, the area of the segment required.

2. 30 — 6 = 24; then 24 × 6 = 144 the square of half the chord, and √144 = 12 rods half the chord; then 6 × 6 = 36 the square of the versed sine; then 144 + 36 = 180 the sum of their squares, and √180 = 13,4164 + rods the length of the diagonal; then 13,-4164 — 12 = 1,4164 rods the difference, and 1,4164 × 5 ÷ 14 = ,506 —; then 13,4164 + ,506 = 13,9224 rods the length of half the arc; then 2)30 = 15 rods the semidiameter, and 13,9224 × 15 = 208,836 square rods the area of the sector, and 15 — 6 = 9 rods the perpendicular of the triangular part of the sector, and 12 rods half the base; therefore 12 × 9 = 108 square rods the area of the triangular part of the sector; then 208,836 — 81 = 127,836 square rods, the area of the segment required.

3. 2)18 = 9 feet the semidiameter; then 9 × 9 = 81,

and 2)12 = 6 half the chord, and 6 × 6 = 36; then 81 — 36 = 45; then √45 = 6,7 +; then 9 — 6,7 = 2,3 feet the versed sine; then 2,3 × 2,3 = 5,29, and 6 × 6 = 36; then 36 + 5,29 = 41,29 the square of the diagonal; then √41,29 = 6,418 — feet length of the diagonal line; 6,418 — 6 = ,418 the difference, and ,418 × 5 ÷ 14 = ,1493; then 6,418 + ,1493 = 6,5673 feet the length of half the arc, and 9 half the diameter; then 6,5673 × 9 = 59,1057 square feet the area of the sector, and 9 — 2,3 = 6,7 the perpendicular of the triangular part of the sector; then 6 × 6,7 = 40,2 the area of the triangular part of the sector; then 59,1057 — 40,2 = 18,9057 square feet, the area of the segment required.

EXAMPLES TO RULE II.

1. 2)40 = 20 half the chord, and 20 × 20 = 400, and 10)400 = 40; then 40 + 10 = 50 the diameter of the circle, and 50)10 = ,2 the tabular versed sine; then from the table the corresponding area is ,111823; then 50 × 50 × 11823 = 279,5575 the area of the segment required.

2. 52)2 = ,0385 the tabular versed sine, and the corresponding area will be ,00995 +; then 52 × 52 × ,00995 = 26,9048 + the area required.

3. 40)10 = ,25 the tabular versed sine; then the area of the tabular segment will be found = to ,153546; then 40 × 40 × ,153546 = 245,6736.

4. 100 ÷ 9 = 09 the tabular segment, and the corresponding area = ,035011; then 100 × 100 × 035011 = 350,11 square chains, which divided by 10 = 35 acres and 1$\frac{76}{100}$ rods, the area required.

5. 2)20 = 10, and 10 × 10 = 100 rods the square of half the chord, and 4 ÷ 100 = 25; then 25 + 4 = 29 rods the diameter of the circle; then 29 ÷ 4 = ,13793 the tabular versed sine, and the corresponding area for the first three figures is ,06476, and the next greater is ,065449; then ,065449 — ,06476 = ,000689, which multiplied by the two remaining figures (93) = ,00064077, which added to ,06476 = ,0654 the area of

the tabular segment; then 29 × 29 × 0654 = 55,0014 rods.

6. 2 ÷ 30 = 15 rods half the chord; then 15 × 15 = 225, and 5)225 = 45, and 45 + 5 = 50 rods the diameter of the circle, and 550)5 = ,1 the tabular versed sine; then ,040875 the tabular area; then 50 × 50 × ,040875 = 102,1875 square rods, the area of the segment required.

PROBLEM XIII.

EXAMPLES.

1. 20 × 20 = 400 the square of the versed sine; then 400)60 = ,15 square chains the tabular area; then the corresponding versed sine ,246 —; then ,246 × 20 = 4,92 chains the versed sine required; then 20 — 4,92 = 15,08, and 15,08 × 4,92 = 74,1936 chains the square of half the chord; then √74,1936 = 8,6135 + chains the half chord, and 8,6135 × 2 = 17,227 + chains, the length of the chord required.

2. 50 × 50 × ,7854 = 1963,5 square chains the area of the circle; then 3)1963,5 = 654,5 square chains the area of the segment to be cut off, and 50 × 50 = 2500 the square of the diameter; then 2500)654,5 = ,2618 chains the tabular area; the corresponding versed sine will be found = to ,36772 +, and ,36772 × 50 = 18,386 chains the versed sine required; then 50 — 18,386 = 31,614, and 31,614 × 18,386 = 581,255 the square of half the chord; then √581,255 = 24,108 chains half the chord; then 24,108 × 2 = 48,216 chains, the length of the chord required.

3. 80 × 80 × ,7854 = 5026,56 the area of the whole circle; then 4)5026,52 = 1256,64 square rods the area of the segment; then 80 × 80 = 6400 the square of the diameter; then 6400)1256,64 = ,19635 the tabular area; the corresponding versed sine will be found = ,298; then ,298 × 80 = 23,84 rods the versed sine required, and 80 — 23,84 = 56,16, and 56,16 × 23,84 = 1338,8544 rods the square of half the chord; then √1338,8544 = 36,59 rods half the required chord; then 36,59 × 2 = 73,18 rods, the chord required.

4. 36 × 36 × ,7854 = 1017,8784 square inches the area of the end of the log; then 3)1017,8784 = 339,-2928 square inches the area of the segment; then 36 × 36 = 1296 the square of the diameter; then 1296)339,-2928 = 0,2618 the tabular area; the corresponding tabular versed sine will be found equal to ,36772; then ,36772 × 36 = 13,23792 inches, the depth required.

5. 60 × 60 × ,7854 = 2827,44, and 5)2827,44 = 565,-488 square rods the area of the proposed segment; then 60 × 60 = 3600, and 3600)565,488 = ,15708 the area of the tabular segment; the corresponding versed sine will be found in the table = to ,254; then ,254 × 60 = 15,24 rods the versed sine of the proposed segment; then 60 — 15,24 = 44,76, and 4476 × 15,24 = 682,1424 the square of half the chord; then $\sqrt{682,1424}$ = 26,118 — half the chord; then 26,118 × 2 = 52,236 rods, the length of chord required.

### PROBLEM XIV.

#### EXAMPLES.

1. 2)12 = 6 the semidiameter; then 6 × 6 = 36 the square of the semidiameter, and 2)8 = 4 the half chord of one of the segments; then 36 — 4 × 4 = 20, and $\sqrt{20}$ = 4,4721; then 12 — 4,4721 = 7,5279 the versed sine of the segment cut off; 12)7,5279 = ,62316 the tabular versed sine; then ,203489 the tabular area; then 12 × 12 = 144 the square of the given diameter, and ,203489 × 144 = 29,3024 the area of one segment; then 12)4,8 = ,4 the tabular versed sine; the corresponding area found in the table is ,293369; then ,293369 × 144 = 42,2451 + the area of the other segment; then 42,2451 + 29,3024 = 71,5475 the area of both segments, and 12 × 12 × ,7854 = 113,0976 the area of the whole circle; then 113,0976 — 71,5475 = 41,5501 the area of the zone required.

2. 25)5 = ,2 the tabular versed sine; then the corresponding tabular area = ,111823, and 25 × 25 × ,111823 = 69,889375 the area of each of the segments;

then 69,889375 × 2 = 139,778750 the area of both segments, and 25 × 25 × ,7854 = 490,875 the area of the whole circle; then 490,875 — 139,77875 = 351,-09625 the area of the middle zone required.

3. 50)18 = ,36 the tabular versed sine, and ,25455 the corresponding area; then 50 × 50 × ,25455 = 636,-375 chains the area of the greater segment, and 50)15 = ,3 the tabular versed sine, and the area corresponding ,198168 square chains; then ,198168 × 50 × 50 = 495,42 the area of the less segment; then 636,375 + 495,42 = 1131,795 the area of both segments, and 50 × 50 × ,7854 = 1963,5 square chains the area of the whole circle; then 1963,5 — 1131,795 = 831,705 square chains, the area of the zone required.

4. 25)10 = ,4 the tabular versed sine, and the area corresponding ,293369; then ,293369 × 25 × 25 = 183,355625 feet the area of one segment, and ,7854 × 25 × 25 = 490,875 square feet the area of the whole circle; then 5)490,875 = 98,175 square feet the area of the less segment, and 183,355625 + 98,175 = 281,-530625 square feet the area of the segments; then 490,875 — 281,530625 = 209,344375 square feet, the area of the zone; then 209,344375 × ,20 = $41,868 + the value of the zone required.

5. 7,584 × 160 = 1256,64 square rods the area of the circle; then 1256,64 ÷ ,7584 = 1600 the square of the diameter of the circle; then √1600 = 40 rods the diameter, and 40 ÷ 4 = 10 rods the versed sine of one segment, and 40 ÷ 5 = 8 rods the versed sine of the other; then 40)10 = ,4 the tabular versed sine of one segment, and 40)8 = ,5 the versed sine of the other; the tabular areas corresponding will be found, the first = to ,293369, and the second = ,392699; then ,293369 + ,392699 = ,686068 the sum of the tabular areas; then ,686068 × 40 × 40 = 1097,7088 square rods the area of both segments, and 1256,64 square rods the area of the whole circle; then 1256,64 — 1097,7088 = 158,9312 square rods the area of the zone; then 158,-9312 × 25 = $39,7328 the value required.

### PROBLEM XV.

EXAMPLES.

1. $20 + 12 = 32$ rods the sum of the two given diameters; $20 - 12 = 8$ the difference; then by the rule, $32 \times 8 \times ,7854 = 201,0624$ square rods the area of the ring required.

2. $24 + 16 = 40$ chains the sum of the two diameters, and $24 - 16 = 8$ chains their difference; then $40 \times 8 \times ,7854 = 251,328$ square chains the area of the ring.

3. $100 \times 10 = 1000$ square chains the given area, and $,7854)1,000 = 1273,236567$ the square of the greater diameter; then $\sqrt{1273,236567} = 35,6824 +$ chains the diameter; then $35,6824 \times \frac{2}{3} = 23,7883 -$ chains the diameter of the less; then $35,6824 + 23,7883 = 59,4707$, and their difference $= 11,8941$; then $59,4707 \times 11,8941 \times ,7854 = 555,55$ the area required.

*Note.*—The above solution is in conformity to the rule given, but it can be more easily solved in the following manner: The diameters being in the proportion of 3 to 2, the squares of those diameters must be in the proportion of 9 to 4; then $9 - 4 = 5$, and $9 : 100 :: 5 : 555,55$ chains, the area of the ring.

4. $3,1416)120 = 38,2 -$ chains the diameter of the greater circle, and 28 the less; then $38,2 + 28 = 66,2$, and $38,2 - 28 = 10,2$; then $66,2 \times 10,2 \times ,7854 = 530,333496$ square chains, the area of the ring; then $530,333496 \times 16 \times ,08 = \$678,828 +$ the value of the ring.

5. All circles are in proportion to each other as the squares of their diameters; therefore $3 + 2 = 5$ the sum of the given proportion, and $3 - 2 = 1$ the difference, and $5 \times 1 \times ,7854 = 3,9270$ chains the area of the given proportion; then $3,927 : 2 \times 2 :: 20$ chains the area given to $20,371785 +$ chains the square of the shorter side; then $\sqrt{20,371785} = 4,5136 -$ chains the diameter of the less circle; then $2 : 4,5136 :: 3 : 6,7704$ chains, the greater diameter.

6. $20 \times 20 \times ,7854 = 314,16$ square rods the area of

the less circle; then $314,16 \times 4 \div 3 = 418,88$ the area of the ring; then $314,16 + 418,88 = 733,04$ square rods the area of the greater circle, and $,7854)73305 = 933,3333$ rods the square of the diameter; then $\sqrt{933,3333} = 30,55 +$ rods, the diameter of the greater circle.

### PROBLEM XVI.

#### EXAMPLES.

1. $2)72 = 36$ half the chord, and $36 \times 36 = 1296$ the square of half the chord, and $30)1296 = 43,2$; then $43,2 + 30 = 73,2$ the diameter of the circle; then $73,2)30 = 0,41$ — the tabular versed sine, and $,303187$ the corresponding area; then $73,2 \times 73,2 \times ,303187 = 1624,5487 +$ the area of the greater segment; then 1296 the square of half the chord; then $20)1296 = 64,8 + 20 = 84,8$ the diameter; then $848)20 = ,23467$ the tabular versed sine, and $,140408 =$ the corresponding area; then $84,8 \times 84,8 \times ,140408 = 1009,6795 +$ the area of the less segment; then $1624,5487 - 1009,6795 = 614,8692$, the area of the lune required.

2. $2 \div 40 = 20$ half the chord; then $20 \times 20 = 400$ the square of half the chord, and $4)400 = 100$; then $100 + 4 = 104$ the diameter of the circle from which the less segment was taken; then $104 \div 4 = ,03846$ the tabular versed sine; the corresponding area is $,00994$; then $104 \times 104 \times ,00994 = 107,511$ the area of the less segment; then 400 the square of half the chord; then $10)400 = 40$, and $40 + 10 = 50$ the diameter of the circle from which the greater segment was taken; then $50)10 = ,2$ the tabular versed sine, and the corresponding area $,111823$; then $,111823 \times 50 \times 50 = 279,5575$ the area of the greater segment; then $279,5575 - 107,511 = 172,0465$, the area of the lune required.

3. $2)24 = 12$ half the chord; then $12 \times 12 = 144$ the square of half the chord; then $10)144 = 14,4$, and $14,4 + 10 = 24,4$ the diameter of the circle from which the greater segment was taken; then $24,4)10 = ,409836$;

the tabular versed sine; the corresponding area is ,303025 +; then ,303025 × 24,4 × 24,4 = 180,408964 the area of the greater segment, and 2)144 = 72, and 72 + 2 = 74 the diameter of the circle from which the less segment was taken; then 74)2 = ,027027 the tabular versed sine, and the corresponding area is = to ,005875; then 74 × 74 × ,005875 = 32,1715 the area of the less segment; then 180,4089 — 32,1715 = 148,-2374 the area of the lune required.

4. 2)48 = 24 the half chord, and 24 × 24 = 576 the square of half the chord; then 18)576 = 32, and 32 + 18 = 50 the diameter of the circle from which the greater segment was taken; then 50)18 = ,36 the tabular versed sine, and the area corresponding is ,25455; then 50 × 50 × ,25455 = 636,375 the area of the greater segment, and 576 the square of half the chord; then 12)576 = 48, and 48 + 12 = 60 the diameter of the circle from which the less segment was taken; then 60)12 = ,2 the tabular segment, and ,111823 the area corresponding; then 60 × 60 × 111823 = 402,5628 the area of the less segment; then 636,375 — 402,5628 = 233,8122 the area of the lune required.

5. 2)50 = 25 half the chord, and 25 × 25 = 625 the square of half the chord, and 18)625 = 34,72 +, and 34,72 + 18 = 52,72 the diameter of the circle from which the greater circle was taken; then 52,72)18 = ,3414 the tabular versed sine, and ,2368 the corresponding area; then ,2368 × 52,72 × 52,72 = 658,1615 + the area of the greater segment, and 625 the square of half the chord; then 15)625 = 41$^2$, and $41\frac{2}{3}$ + 15 = $56\frac{2}{3}$, the diameter of the circle from which the less segment was taken; then $56\frac{2}{3} = \frac{170}{3}$; then $\frac{170}{3}$)15 = ,2647 the tabular versed sine, and ,1664 — the corresponding area; then ,1664 × $\frac{170}{3}$ × $\frac{170}{3}$ = 523,218 — the area of the less segment; then 658,1615 — 523,218 = 134,-9435 the area of the lune.

# SECTION III.

## OF CONIC SECTIONS.

### PROBLEM I.

#### EXAMPLES.

1. $120 - 24 = 96$ the abscissa not given in the question; then $96 \times 24 = 2304$ the rectangle of the two abscissas; then $\sqrt{2304} = 48$ the square root of the product of the two abscissas; then $120 : 40 :: 48 : 16$ the ordinate required.

2. $40 - 24 = 16$, and $16 \times 24 = 384$ the product of the two abscissas, and $\sqrt{384} = 19{,}596$ — the square root of the rectangle of the two abscissas; then $40 : 30 :: 19{,}596 : 14{,}697$ the ordinate required.

3. $50 - 18 = 32$, and $32 \times 18 = 576$, and $\sqrt{576} = 24$; then $50 : 30 :: 24 : 14{,}4$ the ordinate required.

### PROBLEM II.

#### EXAMPLES.

1. $2)40 = 20$ the semi-conjugate; then $20 \times 20 = 400$ the square of the semi-conjugate, and $16 \times 16 = 256$ the square of the ordinate; then $400 - 256 = 144$, and $\sqrt{144} = 12$; then $40 : 120 :: 12 : 36$ the distance from the centre; then $2)120 = 60$; then $60 + 36 = 96$ the greater, and $60 - 36 = 24$ the less abscissa.

2. $2)25 = 12{,}5$, and $12{,}5 \times 12{,}5 = 156{,}25$ the square of the semi-conjugate, and $10 \times 10 = 100$ the square of the ordinate; then $156{,}25 - 100 = 56{,}25$, and $\sqrt{56{,}25} = 7{,}5$; $25 : 35 :: 7{,}5 : 10{,}5$ the distance from the centre; then $2)35 = 17{,}5$, and $17{,}5 + 10{,}5 = 28$ the greater, and $17{,}5 - 10{,}5 = 7$, the less.

3. $2)40 = 20$, and $20 \times 20 = 400$ the square of the

semi-conjugate, and $12 \times 12 = 144$ the square of the ordinate; then $400 - 144 = 256$, and $\sqrt{256} = 16$; then $40 : 60 :: 16 : 24$ the distance from the centre, and $2)60 = 30$, and $30 + 24 = 54$, and $30 - 24 = 6$ the length of the two abscissas required.

### PROBLEM III.

#### EXAMPLES

1. $2)40 = 20$, and $20 \times 20 = 400$ the square of the semi-conjugate, and $16 \times 16 = 256$ the square of the ordinate; then $400 - 256 = 144$, and $\sqrt{144} = 12$; then $20 - 12 = 8$; then $8 : 24 :: 40 : 120$ the transverse diameter required.

2. $2)60 = 30$, and $30 \times 30 = 900$ the square of the semi-conjugate, and $18 \times 18 = 324$ the square of the ordinate; then $900 - 325 = 576$, and $\sqrt{576} = 24$; then $30 - 24 = 6$; then $6 : 24 :: 60 : 240$ the transverse diameter required.

3. $2)60 = 30$, and $30 \times 30 = 900$ the square of the semi-conjugate, and $24 \times 24 = 576$ the square of the ordinate, and $900 - 576 = 324$, and $\sqrt{324} = 18$, and $30 - 18 = 12$; then $12 : 36 :: 60 : 180$ the transverse diameter required.

### PROBLEM IV.

#### EXAMPLES.

1. $120 - 24 = 96$ the greater abscissa, and 24 the less; then $96 \times 24 = 2304$ the rectangle of the two abscissas, and $\sqrt{2304} = 48$ the square root of their product; then $48 : 16 :: 120 : 40$ the conjugate diameter required.

2. $82 - 32 = 50$ the greater abscissa, and $50 \times 32 = 1600$ the rectangle of the two abscissas; then $\sqrt{1600} = 40$ the square root of the product; then $40 : 8 :: 82 : 16{,}4$ the conjugate required.

3. $35 - 7 = 28$ the greater abscissa; then $28 \times 7 = 196$ the rectangle, and $\sqrt{196} = 14$ the square root; then $14 : 10 :: 35 : 25$ the conjugate required.

### PROBLEM V.

EXAMPLES.

1. $24 \times 24 = 576$, and $20 \times 20 = 400$; then $576 + 400 = 976$, and 2)976 = 488 the half sum of the squares of the two diameters; then $\sqrt{488} = 22,09 +$; then $22,09 \times 3,1416 = 69,398$ — the circumference required.

2. $60 \times 60 = 3600$, and $40 \times 40 = 1600$; then $3600 + 1600 = 5200$ the sum of their squares, and 2)5200 = 2600; then $\sqrt{2600} = 51$ —, and $51 \times 3,1416 = 160,2216$ the circumference required.

3. $40 \times 40 = 1600$, and $30 \times 30 = 900$; then $1600 + 900 = 2500$ the sum of their squares; then 2)2500 = 1250, and $\sqrt{1250} = 35,355$; then $35,355 \times 3,1416 = 111\,071 +$

### PROBLEM VI.

EXAMPLES.

1. $60 \times 40 \times ,7854 = 1884,96$ square rods; then 160)1884,96 = 11,7185 acres, the area required.

2. $40 \times 36 \times ,7854 = 1130,976$ square chains, and by dividing by 10 the area will be found = to 113,0976 acres, the area required.

3. $12 \times 4 \times ,7854 \times 30,25 = 1140,4$ square yards required.

### PROBLEM VII.

EXAMPLES.

1. $36 \times 24 = 864$; then $\sqrt{864} = 29,3938$ the diameter required.

2. $46 \times 16 = 784$ the product of the two diameters; then $\sqrt{784} = 28$ rods, the diameter required.

### PROBLEM VIII.

EXAMPLES.

1. 80)20 = ,25 the tabular versed sine, and the corresponding area ,153546; then $,153546 \times 80 \times 50 = 61,4184$ the area required.

2. $35)14 = ,4$ the tabular versed sine, and $,293369$ the corresponding tabular area; then $,293369 \times 35 \times 25 = 256,697875$ square chains, the area required.

3. $60)15 = ,25$ the tabular versed sine, and $,153546$ the corresponding tabular area; then $,153546 \times 60 \times 40 = 368,51 +$ square feet, the area required.

### PROBLEM IX.

#### EXAMPLES.

1. The abscissa is 9; then $\sqrt{9} = 3$ the square root of the less abscissa, and $\sqrt{16} = 4$ the square root of the greater abscissa; then $3 : 6 :: 4 : 8$ the ordinate required.

2. $\sqrt{9} = 3$, and $\sqrt{16} = 4$; then $4 : 8 :: 3 : 6$ the ordinate required, and $3 : 6 :: 4 : 8$ the greater ordinate.

### PROBLEM X.

#### EXAMPLES.

1. $6 \times 6 = 36$ the square of the ordinate, and $2 \times 2 = 4$ the square of the abscissa; then $4 \times \frac{4}{3} = \frac{16}{3} = 5,3333$; then $36 + 5,3333 = 41,3333$, and $\sqrt{41,3333} = 6,4291 +$; then $6,4291 \times 2 = 12,8582$ the length of the arc required.

2. $6 \times 6 = 36$ the square of the given ordinate, and $3 \times 3 = 9$ the square of the abscissa, and $9 \times \frac{4}{3} = 12$; then $36 + 12 = 48$, and $\sqrt{48} = 6,9282 +$; then $6,9282 \times 2 = 13,8564$ the length of the arc required.

### PROBLEM XI.

#### EXAMPLES.

1. $16 \times 12 = 192$; then $192 \times \frac{2}{3} = 128$ the area required.

2. $42 \times 12 = 504$, and $504 \times \frac{2}{3} = 336$ the area required.

3. $8 \times 30 = 240$, and $240 \times \frac{2}{3} = 160$ the area required.

4. $10 \times 18 = 180$, and $180 \times \frac{2}{3} = 120$ the area required.

PROBLEM XII.

EXAMPLES.

1. $10 \times 10 \times 10 = 1000$ the cube of the greater end of the frustrum, and $6 \times 6 \times 6 = 216$ the cube of the less, and $1000 - 216 = 784$ the difference of their cubes, and $10 \times 10 = 100$ the square of the greater end, and $6 \times 6 = 36$ the square of the less; then $100 - 36 = 64$ the difference of their squares; then $64)784 = 12,25$, and $3 \times \frac{2}{3} = 2$; then $12,25 \times 2 = 24,5$ the area of the frustrum required.

2. $24 \times 24 \times 24 = 13824$ the cube of the greater end of the frustrum, and $20 \times 20 \times 20 = 8000$ the cube of the less; then $13824 - 8000 = 5824$ their difference, and $24 \times 24 = 576$ the square of the greater end, and $20 \times 20 = 400$ the square of the less; then $576 - 400 = 176$ the difference of their squares; then $176)5824 = 33,0909 +$, and $6 \times \frac{2}{3} = 4$; then $33,0909 \times 4 = 132,3636$ the area required.

3. $10 \times 10 \times 10 = 1000$ the cube of the greater end of the frustrum, and $6 \times 6 \times 6 = 216$ the cube of the less end; then $1000 - 216 = 784$ the difference of the cubes, and $10 \times 10 = 100$, and $6 \times 6 = 36$; then $100 - 36 = 64$ the difference of their squares; then $64)784 = 12,25$, and $4,2 \times \frac{2}{3} = 2,8$; then $2,8 \times 12,25 = 34,3$ the area required.

## OF THE HYPERBOLA.

PROBLEM I. AND II.

EXAMPLES.

1. $120 + 40 = 160$ the greater abscissa; then $160 \times 40 = 6400$ the rectangle of the two abscissas, and $\sqrt{6400} = 80$ the square root of the product of the two abscissas; then $120 : 72 :: 80 : 48$ the ordinate required.

2. $24 + 8 = 32$ the greater abscissa; then $32 \times 8 = 256$ the rectangle of the two abscissas, and $\sqrt{256} = 16$; then $24 : 21 :: 16 : 14$ the ordinate required.

3. $50 + 12 = 62$ the greater abscissa, and $62 \times 12 =$

744 the rectangle of the two abscissas; then $\sqrt{744} = 27{,}2763$; then $50 : 30 :: 27{,}2763 : 16{,}3658$ — the ordinate required.

PROBLEM III.

EXAMPLES.

1. $2)72 = 36$, and $36 \times 36 = 1296$ the square of the semi-conjugate, and $48 \times 48 = 2304$ the square of the ordinate; then $2304 + 1296 = 3600$ the sum of their squares; then $\sqrt{3600} = 60$ the square root of the sum of the squares of the semi-conjugate and ordinate; then $72 : 120 :: 60 : 100$; then $100 + 60 = 160$ the greater abscissa, and $100 - 60 = 40$ the less.

2. $2)21 = 10{,}5$ the semi-conjugate; then $10{,}5 \times 10{,}5 = 110{,}25$ the square of the semi-conjugate, and $14 \times 14 = 196$ the square of the ordinate, and $196 + 110{,}25 = 306{,}25$ the sum of their squares; then $\sqrt{306{,}25} = 17{,}5$; then $21 : 24 :: 17{,}5 : 20$; then $2)24 = 12$, and $12 + 20 = 32$ the greater abscissa, and $20 - 12 = 8$ the less.

3. $2)36 = 18$ the semi-conjugate, and $18 \times 18 = 324$ the square of the semi-conjugate, and $24 \times 24 = 576$ the square of the ordinate, and $576 + 324 = 900$ the sum of the squares, and $\sqrt{900} = 30$; then $36 : 60 :: 30 : 50$; and $2)60 = 30$; then $30 + 50 = 80$ the greater, and $50 - 30 = 20$ the less.

PROBLEM IV.

EXAMPLES.

1. $80 \times 20 = 1600$ the product of the two abscissas, and $\sqrt{1600} = 40$ the square root of their product; then $40 : 24 :: 60 : 36$ the conjugate diameter required.

2. $48 \times 12 = 576$ the rectangle of the two abscissas, and $\sqrt{576} = 24$ the square root of their product; then $24 : 21 :: 36 : 31{,}5$ the conjugate diameter required.

PROBLEM V.

EXAMPLES.

1. $2)72 = 36$ the semi-conjugate, and $36 \times 36 = 1296$ its square, and $48 \times 48 = 2304$ the square of the

ordinate; then 2304 + 1296 = 3600 the sum of their squares, and $\sqrt{3600} = 60$ the square root of their sum; then 60 + 36 = 96 the sum of the semi-conjugate, and the square root of the sum of the squares of the semi-conjugate and ordinate; then 2304 : 72 × 40 :: 96 : 120 the transverse diameter required.

2. 2)21 = 10,5 the semi-conjugate; then 10,5 × 10,5 = 110,25 the square of the semi-conjugate, and 14 × 14 = 196 the square of its ordinate; then 196 + 110,25 = 306,25 the sum of their squares, and $\sqrt{306,25}$ = 17,5, and 17,5 + 10,5 = 28; then 196 : 21 × 8 :: 28 : 24 the transverse diameter required.

3. 2)36 = 18 the semi-conjugate, and 18 × 18 = 324 its square, and 24 × 24 = 576 the square of the ordinate; then 324 + 576 = 900 the sum of their squares, and $\sqrt{900}$ = 30, and 30 + 18 = 48 the sum of the semi-conjugate, and the square root of the sum of the squares of the semi-conjugate and ordinate; then 576 : 36 × 20 :: 48 : 60 the transverse required.

## PROBLEM VI.

### EXAMPLES.

1. 80 : 60 :: 60 : 45 the perimeter; then 80 × 19 = 1520, and 45 × 21 = 945, and 945 + 1520 = 2465, and 80)2,1637 = ,027046, and ,027046 × 2465 = 66,-66839, and 45 × 15 = 675 fifteen times the perimeter; then 675 + 66,66839 = 741,66839, and 80 × 9 = 720, and 720 + 945 = 1665; then ,027046 × 1665 = 45,-03159, and 45,03159 + 675 = 720,03159; then 720,-03159)741,66839 = 1,03005 —; then 1,03005 × 10 = 10,3005 — length of the arc required.

2. 120 : 72 :: 72 : 43,2 the perimeter; then 19 × 120 = 2280 nineteen times the transverse, and 43,2 × 21 = 907,2 twenty-one times the perimeter, and 120)40 = $\frac{1}{3}$; then 2280 + 907,2 = 3187,2 which multiplied by $\frac{1}{3}$ = 1062,4, and 120 × 9 = 1080 = nine times the transverse; then 1080 + 907,2 = 1987,2 which multiplied by $\frac{1}{3}$ = 662,4, and 43,2 × 15 = 648 = fifteen times the perimeter; then 662,4 + 648 = 1310,4 the divisor, and

1062,4 + 648 = 1710,4 the dividend; then 1310,4)1710,4 = 1,3051 the quotient; then 1,3051 × 48 the ordinate = 62,6448 the length of the curve required.

3. 2)60 = 30 half the conjugate, and 30 × 30 = 900 its square, and 16 × 16 = 256 the square of the ordinate; then 900 + 256 = 1156, and $\sqrt{1156}$ = 34; then 60 : 80 :: 34 : 45$\frac{2}{3}$; then 2)80 = 40 half the transverse; then 45$\frac{2}{3}$ — 40 = 5$\frac{2}{3}$ the less abscissa; then 80 : 60 :: 60 : 45 the perimeter; 19 × 80 = 1520 = nineteen times the transverse diameter, and 45 × 21 = 945 twenty times the perimeter, and 5$\frac{2}{3}$ = $\frac{17}{3}$ the abscissa; then 80)$\frac{17}{3}$ = ,07083; then 1520 + 945 = 2465, and 2465 × ,07083 = 174,59595, and 45 × 15 = 675 fifteen times the perimeter, and 80 × 9 = 720 nine times the transverse, and 720 + 945 = 1665, and 1665 × ,07083 = 117,93195; then 117,93195 + 675 = 792,93195 the divisor, and 174,59595 + 675 = 849,59595 the dividend; then 792,93195)849,59595 = 1,07146, and 1,07146 × 16 = 17,14336 length of the curve from the vertex, and 17,14336 × 2 = 34,28672 the whole length of the curve required.

## PROBLEM VII.

### EXAMPLES.

1. 30 × 10 = 300 the product of the transverse and abscissa, and 10 × 10 × $\frac{5}{7}$ = 71,42857; then 300 + 71,42857 = 371,42857, and $\sqrt{371,42857}$ = 19,2725 —, and 19,2725 × 21 = 404,7225, and 30 × 10 = 300 the product of the transverse by the abscissa, and $\sqrt{300}$ = 17,3205 the square root of their product; then 17,3205 × 4 = 69,282, and 404,7225 + 69,282 = 474,0045, and 474,0045 ÷ 75 = 6,32; then 18 × 10 × 4 = 720 four times the product of the conjugate by the abscissa, and 720 ÷ 30 = 24; then 6,32 × 24 = 151,68 the area required.

2. 100 × 50 = 5000 the product of the transverse by the abscissa; 50 × 50 × $\frac{5}{7}$ = 1785,7142857, and 5000 + 1785,7142857 = 6785,7142857, and $\sqrt{6785,7142857}$ = 82,375447, and 82,375447 × 21 = 1729,884387, and

$100 \times 50 = 5000$ the product of the transverse by the abscissa, and $\sqrt{5000} = 70{,}710678$ the square root of their product, and $70{,}710678 \times 4 = 282{,}842712$ four times the square root of the product of the transverse and abscissa; then $1729{,}884387 + 282{,}842712 = 2012{,}727099$, and $2012{,}727099 \div 75 = 26{,}83636132$, and $60 \times 50 \times 4 = 12000$ four times the product of the conjugate by the abscissa; then $12000 \div 100 = 120$, and $26{,}83636132 \times 120 = 3220{,}3633584$ the area required.

3. $50 \times 25 = 1250$ the product of the transverse by the abscissa, and $30 \times 30 \times \frac{5}{7} = 642{,}857143$, and $1250 + 642{,}857143 = 1892{,}857143$, and $\sqrt{1892{,}857143} = 43{,}506978$; then $43{,}506978 \times 21 = 913{,}646538$, and $25 \times 50 = 1250$, and $\sqrt{1250} = 35{,}355339$; then $35{,}355339 \times 4 = 141{,}421356 =$ four times the square root of the product of the abscissa by the transverse; then $913{,}646538 + 141{,}421356 = 1055{,}067894$, and $1055{,}067894 \div 75 = 14{,}067572$ —, and $30 \times 25 \times 4 = 3000$ four times the product of the conjugate by the abscissa, and $3000 \div 50 = 60$, and $14{,}067572 \times 60 = 844{,}05432$ the area required.

## SECTION IV.

### APPLICATION OF THE MENSURATION OF SUPERFICIES.

#### PROBLEM I.

EXAMPLES.

1. 16 × 16 = 256 chains the area, and 256 ÷ 7854 = 325,94856 +, and √325,94856 = 18,0539 chains, the diameter required.

2. Here 18 × 5,5 = 99 yards the side of the square; then 99 × 99 = 9801 square yards the area; then 9801 ÷ ,7854 = 12479, and √12479 = 111,7094 + yards, the diameter required.

#### PROBLEM II.

EXAMPLES.

1. 24 × 16 = 384 square chains the area of the parallelogram; then 384 ÷ ,7854 = 488,92284 +, and √488,92284 = 22,1116 — chains, the diameter required.

2. 32 × 18 = 576 square rods the area of the parallelogram; then 576 ÷ ,7854 = 733,3842628 —, and √733,3842628 = 27,0816 + rods the diameter required.

#### PROBLEM III.

EXAMPLES.

1. 14 ÷ 2 = 7 rods half the perpendicular; then 17 × 7 = 119 square rods the area, and 119 ÷ ,7854 = 151,5151, and √151,5151 = 12,308 rods; then 12,308 ÷ 4 = 3,077 chains.

2. 16 × 20 = 320 square chains twice the area; then 320 ÷ 2 = 160 the area, and 160 ÷ ,7854 = 203,71785 + the square of the diameter; then √203,71785 =

14,273 — chains; then 14,273 — × 4 = 57,092 — rods, the diameter required.

### PROBLEM IV.

#### EXAMPLES.

1. 18 × 4 = 72 rods the base, and 72 × 72 = 5184 the square of the base, and 6 × 6 = 36 rods the square of the difference; then 5184 — 36 = 5148, and 6 × 2 = 12; then 5148 ÷ 12 = 429 rods the perpendicular of the triangle, and 72 ÷ 2 = 36 half the base, and 429 × 36 = 15444 square rods the area of the triangle; then 15444 ÷ ,7854 = 19663,865546 + the square of the diameter; then √19663,865546 + = 140,2279 +.

2. 24 × 4 = 96 rods length of the base; then 96 × 96 = 9216 the square of the base, and 25 × 25 = 625 the square of the difference between the hypotenuse and perpendicular, and 9216 — 625 = 8591, and 8591 ÷ 625 = 171,82 rods the perpendicular, and 96 ÷ 2 = 48 rods half the base; then 48 × 171,82 = 8247,36 square rods the area; then 8247,36 × 4 = 32989,44; then 32989,44 ÷ ,7854 = 42003,3613 the square of the diameter; then √42003,3613 = 204,9472 rods, the diameter required.

### PROBLEM V.

#### EXAMPLES.

1. Here 14 + 18 + 24 = 56 chains the sum of the three sides; then 56 ÷ 2 = 28 the half sum, and 28 — 24 = 4, and 28 — 18 = 10, and 28 — 14 = 14; then the three differences are 14, 10, and 4; then 28 × 14 × 10 × 4 = 15680 the square of the area, and √15680 = 125,2198 + square chains the area; then 125,2198 ÷ ,7854 = 159,4344 the square of the diameter; then √159,4344 = 12,6267 chains, the diameter required.

2. 18 + 20 + 26 = 64 rods the sum of the three sides; then 64 ÷ 2 = 32 the half sum, and 32 — 26 = 6, 32 — 20 = 12, and 32 — 18 = 14; then the three differences are 14, 12, and 6; then 32 × 14 × 12 × 6 = 32256 the square of the area; then √32256 = 179,6 —

square rods the area; then 179,6 × 3 = 538,8, and 538,8 ÷ ,7854 = 686 the square of the diameter; then √686 = 26,1916 + rods, the diameter required.

PROBLEM VI.

EXAMPLES.

1. 9 : 6 + 7 : : 7 — 6 : 1,4444 + the difference of the segments of the base; then 1,4444 ÷ 2 = ,7222 the half difference, and 9 ÷ 2 = 4,5 chains the half sum; then 4,5 + ,7222 = 5,2222 chains greater of the two segments, and 4,5 — ,7222 = 3,7778 the less; then 6 × 6 = 36 the square of the shorter side, and 3,7778 × 37778 = 14,-27177264 the square of the shorter segment; then 36 — 14,2717 = 21,7283 the square of the perpendicular of the given triangle; then √21,7283 = 4,66 the perpendicular; then 4,66 : 7 : : 6 : 9,01288 — chains, the diameter of a circle which will pass through all the points of the triangle; then 9,01288 ÷ 2 = 4,50644 chains, the distance from each of the angles.

2. Here 60 : 40 + 32 : : 40 — 32 : 9,6 the difference of the segments of the base, and 9,6 ÷ 2 = 4,8 half the difference, and 60 ÷ 2 = 30 the half sum of the base; then 30 + 4,8 = 34,8 rods the greater of the two segments, and 30 — 4,8 = 25,2 the less; then 32 × 32 = 1024 the square of the shorter side, and 25,2 × 25,2 = 635,04 the square of the shorter segment; then 1024 — 635,04 = 388,96 the square of the perpendicular; then √388,96 = 19,722 rods the perpendicular of the triangle; then 19,722 : 40 : : 32 : 64,9 + rods, the diameter required.

PROBLEM VII.

EXAMPLES.

1. 34 × 18 = 612 the rectangle of the two diameters; then √612 = 24,7386 + chains, the diameter required.

2. 18 × 12 × 5 = 1080 the products of the two diameters multiplied by the proportion; then √1080 = 32,86335 + rods, the required diameter.

3. $40 \times 30 = 1200$ square feet the product of the two diameters, and $1200 \times 3 = 3600$ the product of the two diameters multiplied by the given proportion; then $\sqrt{3600} = 60$ feet the diameter of the circle, and $60 \div 3 = 20$ yards, the diameter required.

### PROBLEM VIII.

#### EXAMPLES.

1. $15 \times 15 \times 2 = 450$ twice the square of the given side; then $\sqrt{450} = 21{,}2132 +$ the diameter of the circumscribing circle.

2. $21 \times 21 \times 2 = 888$; then $\sqrt{888} = 29{,}8 -$ chains; then $29{,}8 \times 4 = 119{,}2$ rods, the diameter of the circumscribing circle.

### PROBLEM IX.

#### EXAMPLES.

1. $18 \times 18 = 324$ square chains the square of the diameter; then $324 \div 2 = 162$ the area of the square required; then $\sqrt{162} = 12{,}7279 +$ chains, the side required.

2. $25 \times 10 = 250$ square chains the area of the circle; then $250 \div {,}7854 = 318{,}30914$ chains the square of the diameter; then $318{,}30914 \div 2 = 159{,}15457$ square chains the area of the inscribed square; then $\sqrt{159{,}15457} = 12{,}6156 +$ chains the length of its side, and $250 - 159{,}15457 = 90{,}84543$ square chains, the area of the four segments.

3. $28{,}2744 \div 3{,}1416 = 9$ chains the diameter; then $9 \times 9 = 81$ the square of the diameter, and $81 \div 2 = 40{,}5$ square chains, the area of the inscribed square, $=$ to $4{,}05$ acres.

### PROBLEM X.

#### EXAMPLES.

1. $16 \times 16 = 256$ the square of the longer side, and $12 \times 12 = 144$ the square of the shorter; then $256 + 144 = 400$ the square of the required diameter; then $\sqrt{400} = 20$ feet, the diameter required.

2. Here $64 \times 64 = 4096$ the square of the longer side, and $48 \times 48 = 2304$ the square of the shorter side; then $4096 + 2304 = 6400$ the sum of their squares, and $\sqrt{6400} = 80$ rods the diameter, and $6400 \times ,7854 = 5026,56$ square rods the area, which is = to 31 acres, 1 rood, $26\frac{56}{100}$ perches.

3. $24 \times 24 = 576$ the square of the longer side, and $18 \times 18 = 324$ the square of the shorter side, and $576 + 324 = 900$ the square of the diameter; then $900 \times ,7854 = 706,86$ square chains the area of the circle circumscribing the parallelogram, and $24 \times 18 = 432$ square chains the area of the parallelogram; then $706,86 - 432 = 274,86$ square chains, the area of the segments required.

### PROBLEM XI.

#### EXAMPLES.

1. $\sqrt{3} = 1,73205$ the square root of the number 3; then $1,73205 \times 30 \times \frac{2}{3} = 34,641$ feet, the diameter required.

2. The square root of $3 = 1,73205$; then $1,73205 \times 36 \times \frac{2}{3} = 41,5692$ chains, the diameter required.

3. $1,73205 \times 24 \times \frac{2}{3} = 27,7128$ rods, the diameter required.

### PROBLEM XII.

#### EXAMPLES.

1. $24 \div 2 = 12$ feet the semidiameter of the circle; then $12 \times 3 = 36$; then $36 \div \sqrt{3} = 21,01556 -$ feet, the length of the side required.

2. $36 \div 2 = 18$ rods the semidiameter; then $18 \times 3 = 54$ rods, three times the semidiameter; then $54 \div \sqrt{3} = 31,17635 +$ rods, the length of the side required.

3. $24 \div 3,1416 = 7,63942 -$ chains the diameter of the circle; then $7,63942 \div 2 = 3,81971$ chains the semidiameter, and $3,81971 \times 3 = 11,45913$, and $11,45913 \div \sqrt{3} = 6,61593 +$ chains, the length of the side required.

### PROBLEM XIII.

#### EXAMPLES.

1. Here $10 + 16 + 24 = 50$ feet the length of the

three perpendiculars; then $50 \div \sqrt{3} = 28,86 +$ feet half the length of the side; then $28,86 \times 2 = 57,72$ feet the length of the side; then $57,72 \times \sqrt{3} = 99,973926$, and $99,973926 \times \frac{2}{3} = 66,649 +$ feet, the diameter of the circumscribing circle.

2. $16 + 18 + 24 = 58$ the sum of the three perpendiculars; then $58 \div \sqrt{3} = 33,486 +$ rods half the length of the side; then $33,486 \times 2 = 66,972$ rods the length of the side; then $66,972 \times \sqrt{3} = 115,9988526$, which multiplied by $\frac{2}{3} = 77,3329$ rods, the diameter of the circumscribing circle.

3. $102 \div \sqrt{3} = 58,89 -$ rods half the length of the side of the triangle; then $58,89 \times 2 = 117,78$ rods the length of the side; then $117,78 \times 117,78 \times ,433013 = 6006,81$ square rods the area of the triangle, and $117,78 \times \sqrt{3} \times \frac{2}{3} = 136$ rods the diameter of the circumscribing circle; then $136 \times 136 \times ,7854 = 14526,7584$ square rods the area of the circle; then $14526,7584 - 6006,81 = 8519,9484$ rods, the area of the segments.

## PROBLEM XIV.

### EXAMPLES.

1. $18 \times 12 = 216$ the product of the base by the perpendicular, and $12 + 18 = 30$; then $216 \div 30 = 7,2$ feet, the side of the square.

2. $24 \times 18 = 432$, and $432 \div 24 + 18 = 10,2857 +$ rods, the side of the square required.

## PROBLEM XV.

### EXAMPLES.

1. $1 - ,7854 = ,2146$, and $2 \times 4 + 3 = 11$ roods, and $11 \times 40 + 6 = 446$ square rods the area in the corners; then $,2146 : 1 \times 1 :: 446 : 2078,28518$ rods the square of the diameter required; then $\sqrt{2078,28518} = 45,5882$ rods, the diameter required.

2. $1 - ,7854 = ,2146$ the area left in the corners of a square containing its greatest inscribed circle whose diameter is 1; then $,2146 : 1 \times 1 :: 36,2674 : 169$ the

square of the diameter; then $\sqrt{169} = 13$ rods, the diameter.

3. $1 - ,7854 = ,2146$, and $4 \times 10 = 40$ chains the area left in the corners; then $,2146 : 1 \times 1 :: 40 : 186,3932 +$ the square of the diameter; then $\sqrt{186,3932} = 13,6525$ chains, the diameter required.

### PROBLEM XVI.

#### EXAMPLES.

1. $24 \times 24 = 576$ the square of the diameter, and $576 \times \frac{1}{4} = 144$ the square of the diameter required; then $\sqrt{144} = 12$ chains, the diameter required.

2. $21,75 = 21$ acres 3 roods; $21,75 \times 10 = 217,5$ square chains, and $217,5 \div ,7854 = 276,929 -$ square chains the square of the diameter of the greater circle; and because the less contains $\frac{1}{3}$ of the area of the greater, then $276,929 \times \frac{1}{3} = 92,3096$ chains the square of the less; then $\sqrt{276,929} = 16,638 +$ chains the diameter of the greater, and $\sqrt{92,3096} = 9,607 +$ chains the diameter of the less; then $16,638 - 9,607 + = 7,031$ chains = twice the width of the ring; then $7,031 \div 2 = 3,5155$ chains, the width of the ring.

3. 3 feet 6 inches = 42 inches the diameter in the inside, and 7,5 inches multiplied by $2 = 15$ inches; then $42 + 15 = 57$ inches the whole diameter; then $57 \times 57 = 3249$ the square of the greater diameter, and $42 \times 42 = 1764$ the square of the less; then $3249 - 1764 = 1485$ the difference of their squares, and $1485 \times ,7854 = 1166,319$ square inches the area, and $1166,319 \div 144 = 8,03 -$ square feet; then $8,03 \times 18 =$ \$1,4454 the expense.

### PROBLEM XVII.

#### EXAMPLES.

1. $50 - 30 = 20$ rods the sum of the versed sines of both segments; then $20 \div 2 = 10$ rods the versed sine of each, and $10 \div 50 = ,2$ the tabular versed sine, and ,111823 the corresponding area; then because the segments are equal $,111823 \times 2 = ,223646$ the tabular

area for both segments; then 50 × 50 × ,223646 = 559,115 square rods the area of both segments, and 50 × 50 × ,7854 = 1963,5 square rods the area of one circle; then 1963,5 × 2 = 3927 square rods the area of both; then 3927 — 559,115 = 3367,885 square rods, the area enclosed by their peripheries.

2. 40 — 24 = 16 chains the versed sines of both segments; then 16 ÷ 2 = 8 the versed sine of each, and 40)8 = ,2 the tabular versed sine of each segment, and ,111823 the corresponding area; then ,111823 × 2 = ,223646 square chains the corresponding area for both segments; then 40 × 40 × ,223646 = 357,8336 square chains the area contained in both segments, and 40 × 40 × 7854 = 1256,64 square chains the area of one circle; then 1256,64 × 2 = 2513,28 the area of both circles; then 2513,28 — 357,8336 = 2155,4464 square chains, the area enclosed.

3. 20 — 10 = 10 feet the sum of the two versed sines; then 10 ÷ 2 = 5 the versed sine of each, and 20)5 = ,25 the tabular versed sine of each segment, and ,153546 the tabular area of each; then ,153546 × 2 = ,307092 square feet the tabular area of both segments; then 20 × 20 × ,307092 = 122,8368 square feet the area of both segments, and 20 × 20 × ,7854 = 314,16 square feet the area of each circle; then 314,16 × 2 = 628,32 the area of both circles, and 628,32 — 122,8368 = 505,4832 square feet the area enclosed; 20 — 5 = 15; then 15 × 5 = 75 the square of half the chord of each segment, and √75 = 8,66026 feet half the chord; then 8,66026 × 2 = 17,32052 feet, length of the chord required.

### PROBLEM XVIII.

#### EXAMPLES.

1. 5 — 4 = 1; then 1 : 20 :: 4 : 80 feet; then 80 × 80 = 6400, and 4 ÷ 2 = 2, and 2 × 2 = 4; then 6400 + 4 = 6404 the square of the semidiameter of the circle described by the less wheel; then √6404 = 80,025 — feet the semidiameter of the circle; then 80,025 × 2 = 160,05 feet, the diameter described by the less wheel.

2. 5 — 3 = 2; then 2 : 16 :: 5 : 40; then 40 × 40 = 1600, and 5 ÷ 2 = 2,5 feet, and 2,5 × 2,5 = 6,25 the square of the semidiameter of the greater wheel; then 1600 + 6,25 = 1606,25 the square of the semidiameter of the circle made by the greater wheel; then √1606,25 = 40,078 feet the semidiameter of the circle, and 40,078 × 2 = 80,156 feet, the diameter of the circle required.

3. 4 — 3 = 1 : 30 :: 4 : 120, and 120 × 120 = 14400, and 4 ÷ 2 = 2 the semidiameter of the greater wheel; then 2 × 2 = 4 its square, and 14400 + 4 = 14404 the square of the semidiameter, and √14404 = 120,01666 + feet the semidiameter of the greater circle; then 120,01666 × 2 = 240,03332 feet the diameter, and 30 × 2 = 60; then 240,03332 — 60 = 180,03332 feet the diameter of the less, and 240,0333 ÷ 3 = 80,0111 yards, and 80,111 × 80,111 × ,7854 = 5027,955 — square yards the area of the greater circle, and 180,0333 ÷ 3 = 60,0111 yards the diameter of the less, and 60,0111 × 60,0111 × 7854 = 2828,487 — square yards in the less; then 5027,955 — 2828,487 = 2199,468 square yards, the area of the ring.

PROBLEM XIX.

EXAMPLES.

1. 5280 × 8000 + 20 = 42240020, which multiplied by 20 = 844800400 the square of the distance where a level from the eye strikes the water; then √844800400 = 29065,46 feet, the distance where a level from the eye strikes the water, and 5280 × 8000 + 5280 = 42245280, and 42245280 × 5280 = 223055078400 the square of the distance where a level from the top of the mountain strikes the water; then √223055078400 = 472287 feet the distance; then 472287 + 29065,46 = 501352,46 feet; then 501352,46 ÷ 5280 = 94,95 + miles, the distance required.

2. 5280 × 8000 + 100 = 42240100, and 42240100 × 100 = 4224010000 the square of the distance; then √4224010000 = 64992,3 + feet, the distance, = to 12,309 + miles.

3. $5280 \times 8000 + 100 = 42240100$, and $42240100 \times 100 = 4224010000$, and $\sqrt{4224010000} = 64992,3$ feet the distance where a level from the eye strikes the water, and $5280 \times 8000 + 150 \times 150 = 6336022500$ the square of the distance where a level from the lamp strikes the water, and $\sqrt{6336022500} = 73048,083$ feet the distance; then $73048,083 + 64992,3 = 143040,383$ feet the whole distance between the eye of the observer and the lamp of the lighthouse; then $143040,383 \div 5280 = 27,091$ — miles.

### PROBLEM XX.

#### EXAMPLES.

1. Suppose the side of the square to be 1 yard; then 4 yards the perimeter; then $8 \times 4 = 32$ shillings for enclosing, and 2 shillings for paving; then $2 : 1 :: 32 : 16$ yards, the side of the square required.

2. Suppose the diameter of the circle to be a mile, or 320 rods; then $320 \times 3,1416 = 1005,312$ rods the circumference, and 16 feet 6 inches make one rod; therefore $16 \times 12 + 6 = 198$ inches in one rod; then $198 \times \frac{2}{3} = 132$ the number of dollars the diameters of which would extend the distance of one rod; then $1005,312 \times 132 = 132701,184$ dollars would form a circle whose diameter would be one mile; then $320 \times 320 \times ,7854 \times 33 = 2654023,68$ dollars the value of a circle whose diameter is one mile at the given price; then $132701,184 \div 2654023,68 = ,05$ of a mile the diameter; then $320 \times ,05 = 16$ rods the diameter required; or $2654023,68 : 1 :: 132701,184 : 05$, and $320 \times ,05 = 16$ rods, as before.

3. Suppose the side of the square to be one mile, or 320 rods; then $320 \times 4 = 1280$ rods its perimeter, and 16 feet 6 inches make one rod; then $16 \times 12 + 6 = 198$ inches, and as every cent is to be one inch in diameter, then 198 cents extend the distance of one rod; then $1280 \times 198 = 253440$ cents would extend round the perimeter of a mile square, and 640 the number of acres in one mile square; then $640 \times 40 \times 100 =$

2560000 cents the value of a mile square at the given price; then 2560000 : 320 : : 253440 : 31,68 rods the side of the square and likewise the diameter of the circle; then 31,68 × 3168 = 1003,6224 square rods the area of the square, and 1003,6224 ÷ 160 = 6,27264 acres the square contains; 6,27264 × 40 = 250,9056 dollars the value of the square, and 6,27264 × ,7854 = 4,926531456; then 4,926531456 acres the area of the circle which multiplied by 40 gives 197,06125824 dollars, the value of the circle.

# SECTION V.

## MENSURATION OF SOLIDS.

### PROBLEM I.

EXAMPLES.

1. $18 \times 18 = 324$ inches the area of one side of the given cube; then $324 \times 6 = 1944$ square inches; then $1944 \div 144 = 13{,}5$ square feet, the area required.

2. $25 \times 25 \times 6 = 3750$ square inches; then $3750 \div 144 = 26 + \frac{1}{24}$ square feet.

### PROBLEM II.

EXAMPLES.

1. $2400 \div 6 = 400$ the square of the side; then $\sqrt{400} = 20$ inches, the length of the side required.

2. $24 \div 6 = 4$ square feet the area of the surface of one side; then $\sqrt{4} = 2$ feet, the length of the side required.

### PROBLEM III.

EXAMPLES.

1. $3 \times 3 \times 3 = 27$ solid feet.
2. $32 \times 32 \times 32 \div 1728 = 18\frac{26}{27}$ solid feet.
3. $42 \times 42 \times 42 = 74088$ cubic inches.

### PROBLEM IV.

EXAMPLES.

1. $\sqrt[3]{18} = 2{,}620741$ feet, the length of the side.

2. $2150{,}4252$ solid inches make one bushel; then $2150{,}4252 \times 50 = 107521{,}26$ cubic inches the solidity of the box; then $\sqrt[3]{107521{,}26} = 47{,}5$ inches, the length of the side.

3. $2150{,}4252 \times 10 = 21504{,}52$ — cubic inches the

solidity; then $\sqrt[3]{21504,52} = 27,454 +$ inches, the length of the side required.

### PROBLEM V.

#### EXAMPLES.

1. $8 \times 3 \times 2 = 48$ solid feet, the content required.

2. $36 \times 20 \times 18 = 12960$ solid inches the content; then $12960 \div 1728 = 7,5$ solid feet.

3. Here 5 feet 6 inches $= 66$, and 4 feet $9 = 57$, and 3 feet 9 inches $= 45$ inches; then $66 \times 57 \times 45 = 169290$ solid inches, and $169290 \div 2150,4252 = 78,724$ — bushels.

4. $5,25 \times 2,5 \times 10 = 131,25$ solid feet.

### PROBLEM VI.

#### EXAMPLES.

1. Here two sides are each 10 feet long and 5 feet wide, and the other two are each 10 feet long and 3 feet wide; therefore $10 \times 2 \times 5 = 100$ square feet, and $10 \times 2 \times 3 = 60$ square feet the area of the other two sides; $5 \times 3 \times 2 = 30$ square feet the area of the ends; then $100 + 60 + 30 = 190$ square feet, the area required.

2. Here two faces each 4 feet long and 3 feet wide, and two other faces each 4 feet long and 2,75 feet wide; therefore $4 \times 2 \times 3 = 24$ square feet the area of the two greater faces, and $4 \times 2 \times 2,75 = 22$ square feet the area of the other two sides, and $3 \times 2,75 \times 2 = 16,5$ square feet the area of the two ends; then $24 + 22 + 16,5 = 62,5$ square feet, the whole area required.

3. Here are two sides each 8 feet long, and the breadth of each 4,25 feet, and the other two each 8 feet long and 2,5 feet wide; therefore $8 \times 4,25 + 2 = 68$ square feet, and $8 \times 2,5 \times 2 = 40$ square feet, and $4,25 \times 2,5 \times 2 = 22,25$ square feet the area of the ends; then $68 + 40 + 22,25 = 122,25$ square feet, the area required.

### PROBLEM VII.

#### EXAMPLES.

1. Here are three sides each 12 feet long and 2,5

wide; then $12 \times 2{,}5 \times 3 = 90$ square feet the area of the sides, and $2{,}5 \times 2{,}5 \times 2 \times ,433013 = 5{,}4126 +$ square feet the area of the two ends; then $90 + 5{,}4126 = 95{,}4126 +$ square feet, the area required.

2. Here are three sides each 8 feet long and 14 inches wide; therefore $8 \times 14 \times 3 = 336$, which divided by $12 = 28$ square feet the area of the three sides, and $14 \times 14 \times 2 \times 433013 \div 144 = 1{,}1787 +$ the area of both ends; then $28 + 1{,}1787 = 29{,}1787 +$ square feet the area required.

3. Here are three sides each 9,5 feet long and 2 feet wide; then $9{,}5 \times 2 \times 3 = 57$ square feet the area of the sides, and $2 \times 2 \times 2 \times ,433013 = 3{,}4641$ square feet the area of the ends; then $57 + 3{,}4641 = 60{,}4641$ square feet, the area required.

### PROBLEM VIII.

#### EXAMPLES.

1. $18 \times 18 \times ,433013 \times 40 \div 144 = 38{,}97 +$ solid feet.

2. $18 \times 18 \times ,433013 \times 24 \div 144 = 23{,}386 -$ solid feet.

3. $14 \times 14 \times ,433013 \times 32 \times ,20 \div 144 = \$3{,}772 +$.

4. Here 1 foot 6 inches $= 1{,}5$ feet; then $1{,}5 \times 1{,}5 \times 2{,}598076 \times 16 = 93{,}53 +$ solid feet.

### PROBLEM IX.

#### EXAMPLES.

1. $2 \times 3{,}1416 \times 20 = 125{,}664$ square feet, the convex surface.

2. $3{,}1416 \times 30 \times 5 \div 12 = 39{,}27$ square feet the area of the convex surface, and $30 \times 30 \times 2 \times ,7854 \div 144 = 9{,}8175$ square feet the area of the ends; then $39{,}27 + 9{,}8175 = 49{,}0875$ square feet, the area required.

3. $3{,}1416 \times 16 + 20 \div 12 = 83{,}776$ square feet the area of the convex surface, and $16 \times 16 \times 2 \times ,7854 \div 144 = 2{,}7925$ square feet the area of the ends; then $83{,}776 + 2{,}7925 = 86{,}5685$ square feet the area of the cylinder required.

4. $3{,}1416 \times 4 \times 10 \div 9 = 13{,}9627$ — square yards, the area of the surface required.

PROBLEM X.

EXAMPLES.

1. $16 \times 16 \times {,}7854 \times 20 \div 144 = 27{,}9253 +$ solid feet, the content required.

2. $20 \times 20 \times {,}7854 \times 30 \div 144 = 65{,}45$ solid feet, the content required.

3. $10 \times 10 \times {,}7854 \times 4 \div 144 = 2{,}1816$ solid feet, the content.

4. $15 \times 15 \times {,}7854 \times 16 \div 144 = 19{,}635$ solid feet, the solidity required.

PROBLEM XI.

EXAMPLES.

1. $3{,}1416 \times 3 \times 15 \div 2 = 72{,}686$ square feet, the convex surface of the cone required.

2. $3{,}1416 \times 3 \times 20 \div 2 = 94{,}248$ square feet the convex surface. and $3 \times 3 \times {,}7854 = 7{,}0686$ square feet, the area of the base; then $94{,}248 + 7{,}0686 = 101{,}3166$ square feet, the surface of the cone required.

3. $10 \div 3{,}1416 = 3{,}183 +$ the diameter; then $3{,}183 \div 2 = 1{,}5915$ feet the semidiameter, and $1{,}5915 \times 1{,}5915 + 12 \times 12 = 146{,}53287225$ the sum of the squares of the perpendicular and semidiameter; then $\sqrt{146{,}53287225} = 12{,}105 +$ feet, the length of the slope; then $12{,}105 \times 10 \div 2 = 60{,}525$ square feet, the convex surface required.

PROBLEM XII.

EXAMPLES.

1. 3 feet 6 inches is = to 3,5, therefore $3{,}5 \times 3{,}5 \times {,}7854 \times 3 = 28{,}86345$ solid feet, the content required.

2. $10 \div 3{,}1416 = 3{,}183 +$ feet, the diameter of the base of the cone; then $3{,}183 \times 3{,}183 \times {,}7854 \times 4 = 31{,}829 +$ solid feet, the content required.

3. $6 \div 2 = 3$ feet, the semidiameter of the cone, and $10 + 10 - 3 \times 3 = 91$ the square of the perpendicular

altitude of the cone; then $\sqrt{91} = 9,5394$ feet, the perpendicular altitude of the cone; then $6 \times 6 \times ,7854 \times 9,5394 \div 3 = 89\,9069 +$ cubic feet, the solidity required.

### PROBLEM XIII.

#### EXAMPLES.

1. $15,75 + 22,25 \times 6 \div 2 = 114$ square feet, the convex surface required.

2. $4 + 3 \times 3,1416 \times 5 \div 2 = 54,978$ square feet, the area of the convex surface required.

3. $5 + 4 \times 3,1416 \times 6 \div 2 = 84,8232$ square feet, the area demanded.

### PROBLEM XIV.

#### EXAMPLES.

1. $5 \times 3 = 15$ the product of the top and bottom diameter, and $5 - 3 = 2$ the difference, and $2 \times 2 = 4$ and $4 \div 3 = 1,3333$ one third of the square of the difference; then $15 + 1,3333 = 16,3333$; then $16,3333 \times ,7854 \times 4 = 51,3126 +$ cubic feet, the solidity required.

2. Here 5 feet 6 inches $= 5,5$ feet, and 4 feet 9 inches $= 4,75$ feet, and 7 feet 3 inches $= 7,25$ feet; then $5,5 \times 4,75 = 26,125$ the product of the two diameters, and $5,5 - 4,75 = ,75$, and $,75 \times ,75 \div 3 = ,1875$, one-third of the square of their difference; then $26,125 + ,1875 = 26,3125$, and $26,3125 \times ,7854 \times 7,25 \times 1728 \div 231 \times 63 = 17,79 +$ hogsheads, the solidity required.

3. $5 \times 4,75 = 23,75$ the product, and $5 - 4,75 = ,25$, and $,25 \times ,25 \div 3 = ,0208 +$, one-third of the square of the difference of the two diameters; then $23,75 + ,0208 = 23,7708$, and $23,7708 \times 8 \times ,7854 \times 1728 \div 231 \times 63 = 17,734 +$ hogsheads, the solidity required.

### PROBLEM XV.

#### EXAMPLES.

1. $20 \times 20 \times ,433013 \times 6 \div 144 = 7,2169 -$ cubic feet, the solidity.

2. $2 \times 2 \times \frac{6}{3} = 8$ cubic feet, the solidity required.

3. $30 \times 30 \times 1{,}720477 \times \frac{12}{3} \div 144 = 43{,}012 -$ cubic feet, the solidity required.

4. $20 \times 20 \times 2{,}598076 \times \frac{2{,}75}{3} \div 144 = 23{,}4548 +$ cubic feet, the solidity required.

5. $20 \times 20 \times 4{,}828427 \times \frac{12}{3} \div 144 = 53{,}6492 -$ cubic feet, the solidity required.

### PROBLEM XVI.

#### EXAMPLES.

1. $14 \times 8 = 112$ the product of the two ends, and $14 - 8 = 6$, and $6 \times 6 \div 3 = 12$, one-third of the square of their difference; then $112 + 12 \times {,}433013 \times 10 \div 144 = 3{,}7287 +$ cubic feet, the solidity required.

2. $16 \times 10 = 160$ the product of the two ends, and $16 - 10 = 6$ their difference, and $6 \times 6 \div 3 = 12$, one-third of the square of the difference of the ends; then $160 + 12 \times \frac{8}{3} \div 144 = 9{,}55 +$ cubic feet, the solidity required.

3. Here 18 inches $= 1{,}5$ feet, and 12 inches 1 foot; then $1{,}5 \times 1 = 1{,}5$ the product of the two ends, and $1{,}5 - 1 = {,}5$ the difference of the two ends; then ${,}5 \times {,}5 \div 3 = {,}0833 +$, one-third of the square of the difference of the two ends; then $1{,}5 + {,}0833 = 1{,}5833$, and $1{,}5833 \times 1{,}720477 \times 4{,}5 = 12{,}2581 +$ cubic feet, the solidity required.

4. $10 \times 7 = 70$ the product of the two ends, and $10 - 7 = 3$ the difference, and $3 \times 3 \div 3 = 3$ the third of the square of the difference; then $70 + 3 \times 2{,}598076 \times 5 \div 144 = 6{,}5854 +$ cubic feet, the solidity.

### PROBLEM XVII.

#### EXAMPLES.

1. $5{,}25 \times 2 = 10{,}5$ feet twice the length of the base, and 3,5 length of the edge; then $10{,}5 + 3{,}5 = 14$ feet their sum; then $14 \times 2{,}25 \times {,}75 \div 6 = 3{,}9375$ cubic feet, the solidity required.

2. $35 \times 2 + 55 \times 18 \times 15 \div 6 = 5625$ cubic inches;

then $5625 \div 1728 = 3{,}2552 +$ cubic feet, the solidity of the wedge.

3. $54 + 36 \times 3{,}5 \times 8 \div 6 = 420$, and $420 \div 144 = 2{,}9166 +$ cubic feet, the solidity of the wedge.

PROBLEM XVIII.

EXAMPLES.

1. Here 14 inches the length and 12 inches the breadth of the greater end; then $14 \times 12 = 168$ the area of the greater, and $6 \times 4 = 24$ the area of the less end; then $168 + 24 = 192$ the area of the ends, and $14 + 6 \div 2 = 10$ the length of the middle rectangle, and $12 + 4 \div 2 = 8$ the breadth of the middle rectangle; then $10 \times 8 \times 4 = 320$, four times the area of the middle; then $192 + 320 \times 30{,}5 \div 6 \times 144 = 18{,}074 +$ cubic feet, the solidity required.

2. $27 \times 30 = 810$ square inches the area of the greater end, and $24 \times 18 = 432$ the area of the less, and $30 + 24 \div 2 = 27$ the length of the middle rectangle, and $27 + 18 - 2 = 22{,}5$ breadth of the middle rectangle; then $27 \times 22{,}5 \times 4 = 2430$, four times the area of the middle section; then $2430 + 432 + 810 \times 48 \div 6 \times 144 = 204$ cubic feet, the solidity required.

3. $12 \times 8 = 96$ square inches the area of the greater end, and $8 \times 6 = 48$ square inches the area of the less, and $12 + 8 \div 2 = 10$ inches across the middle, and $8 + 6 \div 2 = 7$ the breadth in the middle; then $10 \times 7 \times 4 = 280$ square inches, four times the area in the middle; then $280 + 48 + 96 \times 5 \div 6 \times 144 = 2{,}4537$ cubic feet, the solidity required.

4. $7 \times 6 = 42$ square feet the area at the top, and $5 \times 3 = 15$ square feet the area at the bottom; then $7 - 5 = 2$, and $6 - 3 = 3$; then $2 \times 3 = 6$ square feet the area at the middle; then $6 \times 4 = 24$ square feet, four times that area; then $42 + 15 + 24 \times 4 \div 6 = 54$ solid feet, the content required.

# SECTION VI.

## OF SURFACES AND SOLIDS.

### PROBLEM I.

#### EXAMPLES.

1. $10 \times 10 \times 3,1416 = 314,16$ square feet, the area of the convex surface.

2. $4 \times 4 \times 3,1416 = 50,2656$ square feet, the area required.

3. $8000 \times 8000 \times 3,1416 = 201062400$ square miles, the area required.

### PROBLEM II.

#### EXAMPLES.

1. $2,5 \times 2,5 \times 2,5 \times ,5236 = 8,181 +$ cubic feet, the solidity required.

2. $3 \times 12 + 4 = 40$ inches the diameter; then $40 \times 40 \times 40 \times ,5236 \div 1728 = 19,3926$ — cubic feet, the solidity required.

3. $8000 \times 8000 \times 8000 \times ,5236 = 268083200000$ cubic miles, the solidity required.

### PROBLEM III.

#### EXAMPLES.

1. $2000 \div ,5236 = 3819,7097$ the cube of the diameter; then $\sqrt[3]{3819,7097} = 15,618 +$ inches, the diameter required.

2. $10 \div ,5236 = 19,09853$ —, the cube of the diameter; then $\sqrt[3]{19,09853} - = 2,67 +$ feet, the diameter required.

3. Here $8 \div ,5236 = 2,481 +$ feet the diameter; then $3,1416 \times 2,481 = 7,7943096$ feet. the circumference required.

### PROBLEM IV.

#### EXAMPLES.

1. $14 \div 3{,}1416 = 4{,}4633 +$ the square of the diameter; then $\sqrt{4{,}4633} = 2{,}1126 +$ feet, the diameter required.

2. 40 square rods = one square rood; then $40 \div 3{,}1416 = 12{,}7323 +$ the square of the diameter; then $\sqrt{12{,}7323} = 3{,}5682 +$ rods, the diameter required.

3. $34 \div 1{,}80 = 18{,}8888 +$ square feet the area of the ball; then $18{,}8888 \div 3{,}1416 = 6{,}01248 -$, the square of the diameter; then $\sqrt{6{,}01248 -} = 2{,}452 +$ feet, the diameter of the ball.

### PROBLEM V.

#### EXAMPLES.

1. $6 \times 3{,}1416 = 18{,}8496$ feet the circumference of the globe; then $18{,}8496 \times 2 = 37{,}6992$ square feet the area of the convex surface of the segment required.

2. $8 \times 3{,}1416 = 25{,}1328$ feet the circumference of the globe given, and $4 \times 4 = 16$ the square of the semidiameter of the globe, and $3 \div 2 = 1{,}5$ the half chord, and $1{,}5 \times 1{,}5 = 2{,}25$ the square of half the chord; then $16 - 2{,}25 = 13{,}75$, and $\sqrt{13{,}75} = 3{,}708 +$ feet; then $4 - 3{,}708 = {,}292$ feet the versed sine of the segment, and ${,}292 \times 25{,}1328 = 7{,}3387 +$ feet, the area of the convex surface required.

3. $30 \div 2 = 15$ inches half the chord; then $15 \times 15 \div 6 = 37{,}5$; then $37{,}5 + 6 = 43{,}5$ inches the diameter of the globe; then $43{,}5 \times 3{,}1416 \times 6 = 820{,}7406$ square inches; then $820{,}7406 \div 144 = 5{,}7 -$ square feet, the convex surface of the segment.

### PROBLEM VI.

#### EXAMPLES.

1. $25 \times {,}31416 \times 4 = 314{,}16$ square inches, the area of the convex surface of the zone required.

2. $30 \div 2 = 15$ inches the semidiameter of the globe, and $16 \div 2 = 8$ the half chord; then $15 \times 15 - 8 \times$

8 = 161, and √161 = 12,7 —, and 15 — 12,7 = 2,3 inches the versed sine of the greater segment, and 10 ÷ 2 = 5 the half chord of the less segment; 15 × 15 — 5 × 5 = 200, and √200 = 14 +, and 15 — 14 = 1 inch the versed sine of the less segment; then 2,3 + 1 = 3,3 inches the sum of the versed sines of both segments; then 30 — 3,3 = 26,7 inches the breadth of the zone, and 30 × 3,1416 = 94,248 inches the circumference of the globe; then 94,248 × 26,7 = 2516,4216 square inches; then 2516,4216 ÷ 144 = 17,4751 + square feet, the area of the zone required.

3. 40 ÷ 2 = 20 inches the semidiameter of the globe, and 16 ÷ 2 = 8 inches half the chord; then 20 × 20 — 8 × 8 = 336, and √336 = 18,3333 +, and 20 — 18,3333 = 1,6667 inches the versed sine of the segment; then 10 + 1,6667 = 11,6667 inches the sum of the versed sines of both segments; then 40 — 11,6667 = 28,3333 inches the breadth of the zone, and 40 × 3,1416 × 28,3333 ÷ 144 = 24,72 + square feet, the area of the convex surface of the zone required.

### PROBLEM VII.

#### EXAMPLES.

1. 30 ÷ 2 = 15 inches the half chord; then 15 × 15 × 3 + 8 × 8 = 739, and 739 × 8 × ,5236 ÷ 1728 = 1,7913 + cubic feet, the solidity required.

2. 40 ÷ 2 = 20 inches half the chord; then 20 × 20 × 3 + 10 × 10 = 1300, and 1300 × 10 × ,5236 ÷ 1728 = 3,93912 cubic feet, the solidity required.

3. 20 ÷ 2 = 10 inches the semidiameter of the bowl; then 10 × 10 × 3 + 6 × 6 = 336, and 336 × 6 × ,5236 ÷ 282 = 3,7432 — beer gallons, the content required.

4. 18 — 3 = 15; then 15 × 3 = 45 the square of half the chord; 45 × 3 + 3 × 3 = 144, and 144 × 3 × ,5236 = 226,1952 cubic inches, the solidity required.

5. 30 ÷ 2 = 15 the semidiameter, and 15 × 15 = 225 the square of the semidiameter; 20 ÷ 2 = 10 inches half the chord, and 10 × 10 = 100 the square of half the chord; then 225 — 100 = 125, and √125

= 11,18 +, and 15 — 11,18 = 3,82 inches the height of the segment, and 100 × 3 + 14,5924 = 314,5924, and 314,5924 × 3,82 × ,5236 = 629,2326 + cubic inches, the solidity required.

### PROBLEM VIII.

#### EXAMPLES.

1. 24 ÷ 2 = 12 the semidiameter of one end, and 20 ÷ 2 = 10 the semidiameter of the other; then 12 × 12 + 10 × 10 = 244, and 9 × 9 ÷ 3 = 27, one-third of the square of the distance of the two ends; then 244 + 27 = 271; then 271 × 3 × 9 × ,5236 = 3831,1812 cubic inches, the solidity required.

2. 5 × 12 = 60 inches the diameter of the globe; then 60 ÷ 2 = 30 inches the semidiameter, and 30 × 30 = 900 its square, and 36 ÷ 2 = 18 inches half the chord, and 18 × 18 = 324 its square, and 900 — 324 = 576, and $\sqrt{576}$ = 24, and 30 — 24 = 6 inches the height of the segment, and 6 + 18 = 24 inches the height of both segments, and 60 — 24 = 36 inches the breadth of the zone, and 60 — 18 = 42, and 42 × 18 = 756 the square of half the chord of the segment whose height was 18 inches; then 756 + 324 = 1080 the sum of the squares of the semidiameters of the two ends of the zone, and 36 × 36 ÷ 3 = 432, one-third of the square of the distance of the two ends; then 1080 + 432 = 1512, and 1512 × 3 × 36 × ,5236 ÷ 1728 = 49,4802 cubic feet, the solidity of the zone required.

3. 3 ÷ 2 = 1,5 feet the semidiameters of each segment; then 1,5 × 1,5 × 2 = 4,5, and 4,2 × 4,2 ÷ 3 = 5,88, one-third of the square of the breadth of the zone; then 4,5 + 5,88 × 3 × 4,2 × ,5236 = 68,48 + cubic feet, the solidity required.

### PROBLEM IX.

#### EXAMPLES.

1. 50 × 40 × 40 × ,5236 ÷ 1728 = 24,2465 + cubic feet, the solidity required.

2. $3 \times 2 \times 2 \times ,5236 = 6,2832$ cubic feet, the solidity of the spheroid.

3. $60 \times 40 \times 40 \times ,5236 \div 1728 = 29,0883 +$ cubic feet, the solidity required.

4. $100 \times 60 \times 60 \times ,5236 \div 1728 = 109,083 +$ cubic feet, the solidity of the spheroid.

#### PROBLEM X.

##### EXAMPLES.

1. $60 \times 100 \times 100 \times ,5236 \div 1728 = 181,8 -$ cubic feet, the solidity of the spheroid.

2. $40 \times 40 \times 30 \times ,5236 \div 1728 = 14,544 +$ cubic feet, the solidity required.

3. $15 \times 20 \times 20 \times ,5236 \div 1728 = 1,818 +$ cubic feet, the solidity required.

#### PROBLEM XI.

##### EXAMPLES.

1. $50 \times 50 \times 2 = 5000$, and $40 \times 40 = 1600$ the square of the diameter of the end; then $5000 + 1600 = 6600$; then $6600 \times 18 \times 2,618 \div 1728 = 18 -$ cubic feet, the solidity required.

2. $(60 \times 60 \times 2 + 36 \times 36) \times 80 \times ,2618 \div 1728 = 102,9746 +$ cubic feet, the solidity required.

3. $(100 \times 100 \times 2 + 80 \times 80) \times 36 \times 2,618 \div 1728 = 143,99$ cubic feet, the solidity required.

#### PROBLEM XII.

##### EXAMPLES.

1. $50 \times 2 \times 30 = 3000$, and $40 \times 24 = 960$; then $3000 + 960 = 3960$, and $3960 \times 18 \times ,2618 \div 1728 = 10,8 -$ cubic feet.

2. $100 \times 2 \times 30 = 6000$ the product of twice the transverse by its conjugate, and $80 \times 48 = 3840$ the product of the transverse and conjugate diameters of each end; then $6000 + 3840 = 9840$ the sum of the products; then $9840 \times 36 \times ,2618 \div 1728 = 53,669$ cubic feet, the solidity required.

3. $100 \times 2 \times 60 = 12000$ twice the transverse by the conjugate, and $60 \times 36 = 2160$ the product of the conjugate and transverse of the ends; then $12000 + 2160 = 14160$ their sum; then $14160 \times 80 \times ,2618 \div 1728 = 171,6244 +$ cubic feet, the solidity required.

### PROBLEM XIII.

#### EXAMPLES.

1. $100 \times 100 = 10000$ the square of the transverse axis, and $60 \times 60 = 3600$, the square of the conjugate; then $3600 \div 10000 = ,36$ the quotient of the square of the revolving axis divided by the square of the fixed axis, and $100 \times 3 = 300$, three times the fixed axis, and $10 \times 2 = 20$ twice the height of the segment, and $300 - 20 = 280$ the difference between three times the fixed axis and twice the height of the segment; then $,36 \times 280 \times 10 \times 10 \times ,5236 = 5277,888$, the solidity required.

2. $50 \times 50 = 2500$ the square of the transverse axis, and $30 \times 30 = 900$ the square of the conjugate; then $900 \div 2500 = ,36$, and $50 \times 3 = 150$, and $5 \times 2 = 10$; then $150 - 10 = 140$; then $,36 \times 140 \times 5 \times 5 \times ,5236 = 659,736$, the solidity required.

3. $100 \times 100 = 10000$ the square of the transverse, and $50 \times 50 = 2500$ the square of the conjugate; then $2500 \div 10000 = ,25$, and $100 \times 3 = 300$, and $12 \times 2 = 24$; then $300 - 24 = 276$; then $,25 \times 276 \times 12 \times 12 \times ,5236 = 5202,4896$, the solidity required.

### PROBLEM XIV.

#### EXAMPLES.

1. $100 \div 50 = 2$ the quotient arising by dividing the fixed axis by the revolving, and $50 \times 3 = 150$, three times the revolving axis, and $12 \times 2 = 24$, twice the height of the segment; then $150 - 24 = 126$ the difference; then $2 \times 126 \times 12 \times 12 \times ,5236 = 19000,3968$, the solidity required.

2. $50 \div 30 = 1\frac{2}{3}$, the quotient arising by dividing the fixed axis by the revolving, and $30 \times 3 = 90$, three

times the revolving axis, and $6 \times 2 = 12$, twice the height of the segment; then $90 - 12 = 78$ their difference; then $1\frac{1}{3} = \frac{4}{3}$; then $\frac{4}{3} \times 78 \times 6 \times 6 \times ,5236 = 2450,448$, the solidity required.

### PROBLEM XV.

#### EXAMPLES.

1. $48 \times 48 \times ,7854 = 1809,5616$ the area of the circular base, and $84 \div 2 = 42$ half the height; then $1809,5616 \times 42 = 7600,15872$ the solidity required.
2. $100 \times 100 \times ,7854 \times 25 = 196350$ the solidity required.
3. $100 \times 100 \times ,7854 \times 15 = 117810$ the solidity required.
4. $40 \times 40 \times ,7854 \times 15 = 18849,6$ the solidity required.

### PROBLEM XVI.

#### EXAMPLES.

1. $58 \times 58 = 3364$ the square of the greater end, and $30 \times 30 = 900$ the square of the less, and $3364 + 900 = 4264$ the sum of their squares; then $4264 \times 18 \times ,3927 = 30142,4104$ the solidity required.
2. $60 \times 60 = 3600$ the square of the greater end, and $50 \times 50 = 2500$ the square of the less, and $2500 + 3600 = 6100$ the sum of the squares of the two ends; then $6100 \times 10 \times ,3927 = 23954,7$ the solidity required.

### PROBLEM XVII.

#### EXAMPLES.

1. $12 \times 12 = 144$ the square of the semidiameter, and $15,8745 \times 15,8745 = 251,99975 +$, the square of the middle diameter; then $251,99975 + 144 \times 10 \times ,5236 = 2073,454691$ the solidity required.
2. $52 \times 52 = 2704$ the square of the radius of the base, and $68 \times 68 = 4624$ the square of the middle diameter; then $2704 + 4624 \times 50 \times ,5236 = 191847,04$ the solidity required.

### PROBLEM XVII.

#### EXAMPLES.

1. 32 ÷ 2 = 16, and 16 × 16 = 256 the square of the greater semidiameter, and 24 ÷ 2 = 12, and 12 × 12 = 144 the square of the less, and 28,1708 × 28,1708 = 793,5939 +, the square of the middle diameter; then 793,5939 + 144 + 256 = 1193,5939 the sum of the squares; then 1193,5939 × 20 × ,5236 = 12499,31532 the solidity required.

2. 96 ÷ 2 = 48 the greater diameter, and 48 × 48 = 2304 its square, and 54 ÷ 2 = 27 the less, and 27 × 27 = 729 the square of the less semidiameter, and 76,424392 × 76,424392 = 5840,6876 + the square of the middle diameter; then 5840,6876 + 2304 + 729 × 25 × ,5236 = 116156,57 + the solidity required.

3. 10 ÷ 2 = 5, and 5 × 5 = 25 the square of the greatest semidiameter, and 6 ÷ 2 = 3, and 3 × 3 = 9 the square of the less, and 8,5 × 8,5 = 72,25; then 72,25 + 9 + 25 = 106,25 the sum of their squares; then 106,25 × 12 × ,5236 = 667,59 cubic inches, the solidity.

4. 8 ÷ 2 = 4, and 4 × 4 = 16 the square of the greater semidiameter, and 6 × 6 = 36 the square of the middle diameter; then 36 + 16 × 10 × ,5236 = 272,272 the solidity required.

## SECTION VII.

## MENSURATION OF SOLIDS.

### PROBLEM I.

EXAMPLES.

1. $2150,4252 \times 5 = 10752,1260$ cubic inches in five bushels; then $\sqrt[3]{10752,126} = 22,07 +$ inches, length of the side required.

2. $\sqrt[3]{282} = 6,5576 +$ inches, the length of the side.

### PROBLEM II.

EXAMPLES.

1. $20 \times 20 = 400$ the square of one side, and the length, breadth, and thickness are each equal; therefore $400 \times 3 = 1200$ the square of the diagonal; then $\sqrt{1200} = 34,641 +$ inches, the length of the diagonal required.

2. $\sqrt[3]{3834} = 15,6513 +$ inches the side of the cube; then $15,6513 \times 15,6513 \times 3 = 734,88957507$ the square of the diagonal; then $\sqrt{734,88957507} = 27,1088 +$ inches, the length of the diagonal required.

### PROBLEM III.

EXAMPLES.

1. 18 inches $= 1,5$ feet; then $1,5 \times 40 = 60$, and $80 \div 60 = 1\frac{1}{3}$ feet $= 20$ inches, the thickness required.

2. $14 \times 20 = 280$, and $36 \times 144 = 5184$; then $5184 \div 280 = 18,51425$ inches, the breadth required.

### PROBLEM IV.

EXAMPLES.

1. $20 \times 20 = 400$, and $40 \times 144 = 5760$; then $5760 \div 400 = 14,4$ feet, the length required.

2. $18 \times 14 = 252$ the product of the breadth and thickness; then $144 \times 30 = 4320$ the product of the solidity by the number of square inches in a square foot; then $4320 \div 252 = 17{,}1428 +$ feet, the length of the stick required.

### PROBLEM V.

#### EXAMPLES.

1. $20 \times 20 \times ,7854 = 314{,}16$ the divisor, and $144 \times 20 = 2880$ the dividend; then $2880 \div 314{,}16 = 9{,}1673 +$ feet, the length required.

2. $5{,}5 \times 5{,}5 \times ,7854 = 23{,}75835$ the divisor, and $20 \times 63 \times 231 = 291060$ cubic inches the solidity; then $291060 \div 23{,}75835 = 12251{,}27 +$; then $12251{,}27 \div 1728 = 7{,}09 -$ feet, the depth required.

3. $15 \times 15 \times ,7854 = 176{,}715$, and $24 \times 144 = 3456$ the dividend; then $3456 \div 176{,}715 = 19{,}557 -$; then $30 - 19{,}557 = 10{,}443 +$ feet, the length required.

### PROBLEM VI.

#### EXAMPLES.

1. $30 \times ,7854 = 23{,}562$ the divisor, and $60 \times 144 = 8640$ the dividend; then $8640 \div 23{,}562 = 370{,}9362 +$ the square of the diameter; then $\sqrt{370{,}9362} = 19{,}26 -$ inches, the diameter required.

2. $,7854 \times 8 = 6{,}2832$ the divisor; then $2150{,}4252 \div 6{,}2832 = 342{,}25$ the square of the diameter; then $\sqrt{342{,}25} = 18{,}5$ inches, the diameter required.

3. $7{,}5 \times ,7854 = 5{,}8905$ the diameter, and $2150{,}4252 \div 2 = 1025{,}2126$ cubic inches the solidity of a half bushel; then $1025{,}2126 \div 5{,}8905 = 174{,}045 -$ the square of the diameter; then $\sqrt{174{,}045} = 13{,}1926 +$ inches, the diameter required.

### PROBLEM VII.

#### EXAMPLES.

1. $24 \div 2 = 12$ the semidiameter, and $12 \times 12 \times 2 = 288$, twice the square of the semidiameter; then $288 \times 25 \div 144 = 50$ cubic feet, the solidity.

2. $20 \times 20 \times ,7854 \times 40 \div 144 = 87,2666 +$ cubic feet the solidity of the cylinder, and $20 \div 2 = 10$ inches the semidiameter; then $10 \times 10 \times 2 \times 40 \div 144 = 55,5555 +$ cubic feet the solidity when hewn square; therefore $87,2666 - 55,5555 = 31,7111$ cubic feet, the solidity of the chips.

3. $18 \div 2 = 9$, and $9 \times 9 \times 2 = 162$, twice the square of the semidiameter, and $24 \times 144 = 3456$, and $3456 \div 162 = 21\frac{1}{3}$ feet, or 21 feet 4 inches, the length required.

PROBLEM VIII.

EXAMPLES.

1. $15 \div 3 = 5$ feet, one-third of the altitude of the cone; then $,7854 \times 5 = 3,927$ the divisor, and $30 \times 144 = 4320$; then $4320 \div 3,927 = 1100 +$, the square of the diameter; then $\sqrt{1100} = 33,1647 +$ inches, the diameter of the base required.

2. $9 \div 3 = 3$ feet, one-third of the altitude of the cone; then $,7854 \times 3 = 2,3562$ the divisor, and $16 \times 144 = 2304$, and $2304 \div 23562 = 977,8 +$ the square of the diameter; then $\sqrt{977,8} = 31,27 -$ inches, the diameter required.

3. $,7854 \times \frac{8}{3} = 2,0944$ the divisor; $18 \times 144 = 2592$; then $2592 \div 2,0944 = 1237,5716 +$; then $\sqrt{1237,5716} = 35,18 -$ inches, the diameter required.

PROBLEM IX.

EXAMPLES.

1. $20 \times 20 \times ,7854 = 314,16$ the divisor, and $20 \times 144 = 2880$, and $2880 \div 314,16 = 9,1673 +$, one-third of the altitude of the cone; then $9,1673 \times 3 = 27,5019$ feet, the altitude.

2. $2 \times 2 \times ,7854 = 3,1416$ the divisor, and $30 \div 3,1416 = 9,5492 +$ feet, one-third of the altitude; then $9,5492 \times 3 = 28,6476$ feet, the altitude required.

PROBLEM X.

EXAMPLES.

1. $6 \times 6 \times 6 \div 2 = 108$, and $\sqrt[3]{108} = 4,7622 +$, the

altitude left at the vertex, and $6 - 4{,}7622 = 1{,}2378$ feet, the height of the other.

2. $10 \times 10 \times 10 \div 3 = 333{,}3333 +$; then $333{,}3333 \times 2 = 666{,}6666$, and $\sqrt[3]{666{,}6666} = 8{,}7358 +$ feet the altitude left after the lower section is cut off; then $10 - 8{,}7358 = 1{,}2642$ feet the height of the lower section, and $\sqrt[3]{333{,}3333 +} = 6{,}9336 +$ feet the altitude left at the vertex after the middle section is cut off; then $8{,}7358 - 6{,}9336 = 1{,}8022$ feet the height of the middle section, and the various heights are 6,9336, 1,8022, and 1,2642 feet.

3. $12 \times 12 \times 12 = 1728$ the cube of the altitude, and $3 + 4 + 5 = 12$, and $12 - 5 = 7$; then $12 : 1728 :: 7 : 1008$ the cube of the altitude left at the vertex after the lower section is cut off; then $\sqrt[3]{1008} = 10{,}0266 -$ feet the altitude; then $12 - 10{,}0266 = 1{,}9734$ feet the height of the lower section; then $12 : 1728 :: 3 : 432$ the cube of the altitude left at the vertex after the second section is cut off, and $\sqrt[3]{432} = 7{,}5595 +$ feet the altitude; then $10{,}0266 - 7{,}5595 = 2{,}4671$ feet the height of the middle section; then the heights of the sections are 7,5595 +, 2,4671, and 1,9734 feet, the altitudes required.

## PROBLEM XI.

### EXAMPLES.

1. $4 \times 4 \times 2 = 32$ the dividend; $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ or ,01 the divisor; then $32 \div ,01 = 3200$ inches the length of the wire; then $3200 \div 36 = 88{,}888 +$ yards, the length of the wire required.

2. $3 \times 3 \times 3 = 27$ the square of the base multiplied by one-third of the altitude, and $,001 \times ,001 = ,000001$ the square of the diameter of the wire; then $27 \div ,000001 = 27000000$ inches the length of the wire; then $27000000 \div 36 = 750000$ yards, and $750000 \div 1760 = 426\frac{3}{22}$ miles, the length of the wire required.

3. $4 \times 4 \times 3 = 48$ the square of the base multiplied by one-third of the altitude, and $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$ the square of the diameter of the wire; then $48 \div \frac{1}{64} = 3072$ inches the length of the wire, and $3072 \div 12 = 256$ feet, the length of the wire required.

# SECTION VIII.

## OF REGULAR BODIES.

### PROBLEM I.

EXAMPLES.

1. $5 \times 5 \times 5 \times \frac{1}{12} \times 1,4142 = 14,73125$ the solidity required.
2. $6 \times 6 \times 6 \times \frac{1}{12} \times 1,4142 = 25,4556$ the solidity required.
3. $4 \times 4 \times 4 \times \frac{1}{12} \times 1,4142 = 7,5424$ cubic feet, the solidity required.

### PROBLEM II.

EXAMPLES.

1. $9,46 \times 12 \div 1,4142 = 80,27153 +$ the cube of the side; then $\sqrt[3]{80,27153} = 4,314 -$ feet, the length of the side required.
2. $25,452 \times 12 \div 1,4142 = 216 -$ the cube of the side; then $\sqrt[3]{216} = 6$ feet, the length of the side.
3. $36 \times 12 \div 1,4142 = 305,473 +$ the cube of the side; then $\sqrt[3]{305,473} = 6,734 +$ feet, the length of the side required.

### PROBLEM III

EXAMPLES.

1. $6 \times 6 \times 6 \times \frac{1}{3} \times 1,4142 = 101,8224$ cubic feet, the solidity required.
2. $5 \times 5 \times 5 \times \frac{1}{3} \times 1,4142 = 58,925$ cubic inches, the solidity required.
3. $8 \times 8 \times 8 \times \frac{1}{3} \times 1,4142 = 241,3568$ cubic feet, the solidity required.

### PROBLEM IV.

EXAMPLES.

1. $32 \times 3 \div 1,4142 = 67,8829$ the cube of the side;

then $\sqrt[3]{67,8829} = 4,0793 +$ feet, the length of the side required.

2. $300 \times 3 \div 1,4142 = 636,4022 +$ the cube of the side; then $\sqrt[3]{636,4022} = 8,6 +$ feet, the length of the side required.

3. $60 \times 3 \div 1,4142 = 127,8 +$ the cube of the side; then $\sqrt[3]{127,8} = 5,032 +$ feet, the length of the side required.

### PROBLEM V.

#### EXAMPLES.

1. $4 \times 4 \times 4 \times 5 \times 1,53262 = 490,4384$ cubic feet, the solidity required.

2. $6 \times 6 \times 6 \times 5 \times 1,53262 = 1655,2296$ cubic inches, the solidity required.

3. $2 \times 2 \times 2 \times 5 \times 1,53262 = 61,3048$ cubic feet, the solidity required.

### PROBLEM VI.

#### EXAMPLES.

1. $206,901 \div 1,53262 \times 5 = 27$ the cube of the side; then $\sqrt[3]{27} = 3$ inches, the length of the side required.

2. $600 \div 1,53262 \times 5 = 78,297295$ the cube of the side; then $\sqrt[3]{78,297295} — = 4,278$ feet, the side required.

3. $7,6631 \div 1,53262 \times 5 = 1$ the cube of the side, and $\sqrt[3]{1} = 1$ foot, the length of the side required.

### PROBLEM VII.

#### EXAMPLES.

1. $\sqrt{5} \times 3 + 7 = 13,708204$, which divided by 2 gives a quotient = to 6,854102, the square root of which is 2,61803; then $2,61803 \times 3 \times 3 \times 3 \times \frac{5}{6} = 58,905675$, the solidity required.

2. Here $2,61803 \times 4 \times 4 \times 4 \times \frac{5}{6} = 139,62826$ cubic feet, the solidity required.

3. $2,61803 \times \frac{5}{6} = 2,181691$, the solidity required.

### PROBLEM VIII.

#### EXAMPLES.

1. $\sqrt{5} = 2,236068$ the square root of the number 5;

then $2,236068 \times 3 + 7 \div 2 = 6,854102$, and $\sqrt{6,854102} = 2,61803$; then $2,61803 \times \frac{5}{6} = 2,181691$; then $58,905675 \div 2,181691 = 27$ the cube of the side; then $\sqrt[3]{27} = 3$ inches, the side required.

2. $600 \div 2,181691 = 275,016$ the cube of the side required; then $\sqrt[3]{275,016} = 6,503 +$ feet, the side.

3. $\sqrt{5} = 2,236068$, which multiplied by $3 = 6,708204$, and $6,708204 + 7 \div 2 = 6,854102$, and $\sqrt{6,854102} = 2,18691$; then $2,18691 \div 2,18691 = 1$ the cube of the side, and $\sqrt[3]{1} = 1$, the length of the side required.

## SECTION IX.

### OF REGULAR BODIES.

#### PROBLEM I.

EXAMPLES.

1. $4 \times 4 \times 1{,}73205 = 27{,}7128$, the area required.

2. $6 \times 6 \times 1{,}73205 = 62{,}3538$ square feet, the area required.

3. $8 \times 8 \times 1{,}73205 = 110{,}8512$ square inches, the area.

#### PROBLEM II.

EXAMPLES.

1. $8 \div 1{,}73205 = 4{,}67654$ — the square of the side; then $\sqrt{4{,}67654} = 2{,}1625 +$ feet, the length of the side.

2. $12 \div 1{,}73205 = 6{,}927629 +$ the square of the side; then $\sqrt{6{,}927629} = 2{,}632 +$ feet, the length of the side.

3. $20 \div 1{,}73205 = 11{,}547 +$ the square of the side; then $\sqrt{11{,}547} + = 3{,}398 +$ inches, the length of the side.

#### PROBLEM III.

EXAMPLES.

1. $8 \times 8 \times 6 = 384$ square inches, the area required.

2. $10 \times 10 \times 6 = 600$ square inches, the area.

#### PROBLEM IV.

EXAMPLES.

1. $216 \div 6 = 36$ the square of the side; then $\sqrt{36} = 6$ feet, the length of the side required.

2. $300 \div 6 = 50$ the square of the side; then $\sqrt{50} = 7{,}071 +$ feet, the side required.

#### PROBLEM V.

EXAMPLES.

1. $3 \times 3 \times 3{,}4641 = 31{,}1769$ square feet, the area required.

2. $4,5 \times 4,5 \times 3,4641 = 70,148 +$ square feet, the area required.

3. $6 \times 6 \times 3,4641 = 124,7076$ square feet, the area required.

PROBLEM VI.

EXAMPLES.

1. $20 \div 3,4641 = 5,7735 +$ the square of the side; then $\sqrt{5,7735} = 2,4$ feet, the length of the side.

2. $36 \div 3,4641 = 10,39231 -$ the square of the side required; then $\sqrt{10,39231} = 3,2237 +$ feet, the side.

3. $24 \div 3,4641 = 6,9282 +$ the square of the side; then $\sqrt{6,9282} + = 2,6321 +$ feet, the length of the side.

PROBLEM VII.

EXAMPLES.

1. $6 \times 6 \times 20,64578 = 743,24808$ square feet, the area of the surface required.

2. $5,5 \times 5,5 \times 20,64578 = 624,534845$ square feet, the area required.

3. $3 \times 3 \times 20\,64578 = 185,812 +$ square feet, the area required.

PROBLEM VIII.

EXAMPLES.

1. $840 \div 20,64578 = 40,686232 +$ the square of the side; then $\sqrt{40,686232} = 6,3785 +$ feet, the length of the side required.

2. $150 \div 20,64578 = 7,2654$ the square of the side; then $\sqrt{7,2654} + = 2,6954 +$ yards, the length of the required side.

3. $120 \div 20,64578 = 5,812326 -$ the square of the side; and $\sqrt{5,812326} = 2,4108 +$ feet, the length of the side.

PROBLEM IX.

EXAMPLES.

1. $6 \times 6 \times 8,66025 = 311,769$ square feet, the area required.

2. $4,5 \times 4,5 \times 8,66025 = 175,37 +$ square feet, the required area.

3. $3 \times 3 \times 8,66025 = 77,94225$ square feet, the area.

## PROBLEM X.

### EXAMPLES.

1. 294 ÷ 866025 = 33,9482 + the square of the side; then $\sqrt{33,9482}$ = 5,8265 + feet, the length of the side required.

2. 16 ÷ 8,66025 = 1,8475 + the square of the side; then $\sqrt{1,8475}$ = 1,36 — yards, the length of the side.

3. 284 ÷ 8,66025 = 32,7935 + the square of the side; then $\sqrt{32,7935}$ = 5,7265 + feet, the length of the side.

# SECTION X.

## MENSURATION OF REGULAR BODIES.

### PROBLEM I.

EXAMPLES.

1. $7 \times 7 \times 7 \times ,11785 = 40,42255$ cubic feet, the solidity.

2. $3,5 \times 3,5 \times 3,5 \times ,11785 = 5,05281875$ cubic feet, the solidity required.

3. $5 \times 5 \times 5 \times ,11785 = 14,73125$ cubic feet, the solidity required.

### PROBLEM II.

EXAMPLES.

1. $230 \div ,11785 = 1951,634$ the cube of the side; then $\sqrt[3]{1951,634} = 12,44 +$ feet, the length of the side required.

2. $128 \div ,11785 = 1086,126$ the cube of the side; then $\sqrt[3]{1086,126} = 10,279 +$ feet, the length of the side.

3. $3 \div ,11785 = 25,456$ the cube of the side; then $\sqrt[3]{25,456} = 2,941 +$ feet, the length of the side

### PROBLEM III.

EXAMPLES.

1. $12 \times 3 \div 11 = 3,2727 +$; then $12 - 3,2727 = 8,7273 -$ inches, the perpendicular altitude required.

2. $5 \times 3 \div 11 = 1,3636 +$, and $5 - 1,3636 = 3,6364$ feet, the altitude required.

### PROBLEM IV.

EXAMPLES.

1. $30 \times 3 \div 11 = 8,1818 +$; then $30 - 8,1818 = 21,8182$; $21,8182 \times 8,1818 \times 4 = 714,0486 -$ the

square of the side; then $\sqrt{714,0486} = 26,72$ inches, the length of the side required.

2. $22 \times 3 \div 11 = 6$, and $22 - 6 = 16$; then $16 \times 6 \times 4 = 384$, and $\sqrt{384} = 19,59 +$ inches, the length of the side required.

3. $11 \times 3 \div 11 = 3$, and $11 - 3 = 8$; then $8 \times 3 \times 4 = 96$, and $\sqrt{96} = 9,798 -$ inches, the length of the side.

### PROBLEM V.

#### EXAMPLES.

1. Suppose the diameter 5,5; then $5,5 \times 3 \div 11 = 1,5$, and $5,5 - 1,5 = 4$; then $4 \times 1,5 \times 4 = 24$ the square of the side of the greatest tetraedron that can be made from a globe whose diameter = the one supposed; then $24 \times \sqrt{24} \times ,11785 = 13,85916 -$ cubic inches, the solidity of the greatest tetraedron that can be formed within the supposed globe; then $10 \times 10 \times 10 \times ,11785 = 117,85$ cubic inches, the solidity of the tetraedron whose side is given; then $13,85916 : 5,5 \times 5,5 \times 5,5 :: 117,85 : 1414,753413 +$ the cube of the diameter required; then $\sqrt[3]{1414,753413} = 11,226 +$ inches, the diameter.

2. Suppose the diameter to be 11 inches; then $11 \times 3 \div 11 = 3$; $11 - 3 = 8$; then $8 \times 3 \times 4 = 96$ the square of the side of the greatest tetraedron that can be formed within its periphery; then $96 \times \sqrt{96} \times ,11785 = 110,87328$ cubic inches its solidity, and $6 \times 6 \times 6 \times ,11785 = 25,4556$ cubic inches, the solidity of the tetraedron whose side is given; then $110,87328 : 11 \times 11 \times 11 :: 25,4556 : 305,496 +$ the cube of the required diameter; then $\sqrt[3]{305,496} = 6,735 +$ inches, the diameter required.

3. Suppose the diameter to have been 22 inches; then $22 \times 3 \div 11 = 6$, and $22 - 6 = 16$; then $16 \times 6 \times 4 = 384$ the square of the side of the greatest equilateral triangular pyramid that can be formed within the periphery of the supposed globe; then $384 \times \sqrt{384} \times ,11785 = 889,584968$ cubic inches the solidity, and 24

$\times$ 24 $\times$ 24 $\times$ ,11785 = 1629,1584 cubic inches, the solidity of the tetraedron whose side is given; then 889,5849 : 22 $\times$ 22 $\times$ 22 : : 1629,1584 : 19500 the cube of the diameter required; then $\sqrt[3]{19500}$ = 26,91 + inches, the diameter required.

### PROBLEM VI.

EXAMPLES.

1. 3,25 $\times$ 3,25 $\times$ 3,25 $\times$ ,4714 = 16,1822 + cubic feet, the solidity required.

2. 2 $\times$ 2 $\times$ 2 $\times$ ,4714 = 3,7712 cubic inches, the solidity required.

3. 5 $\times$ 5 $\times$ 5 $\times$ ,4714 = 58,925 cubic feet, the solidity required.

### PROBLEM VII.

EXAMPLES.

1. 12 $\div$ ,4714 = 25,456 + the cube of the side; then $\sqrt[3]{25{,}456}$ = 2,9416 + feet, the side required.

2. 128 $\div$ ,4714 = 271,5316 + the cube of the side; then $\sqrt[3]{271{,}5316}$ = 6,475 + feet, the length of the side required.

3. 9 $\div$ ,4714 = 19,092 + the cube of the side; then $\sqrt[3]{19{,}092}$ = 2,6727 + feet, the side required.

### PROBLEM VIII.

EXAMPLES.

1. 2 $\times$ 2 $\times$ 2 $\times$ 7,66312 = 61,30496 cubic feet, the solidity required.

2. 3 $\times$ 3 $\times$ 3 $\times$ 7,66312 = 206,90424 cubic feet, the solidity.

3. 4 $\times$ 4 $\times$ 4 $\times$ 7,66312 = 490,43968 cubic inches, the solidity required.

### PROBLEM IX.

EXAMPLES.

1. 500 $\div$ 7,66312 = 65,247575 + the cube of the side; then $\sqrt[3]{65{,}247575}$ = 4,026 — feet, the length of the side required.

2. $128 \div 7{,}66312 = 16{,}7 +$ the cube of the side; then $\sqrt[3]{16{,}7} = 2{,}556 +$ feet, the length of the side required.

3. $1728 \div 7{,}66312 = 225{,}49562$ the cube of the side; then $\sqrt[3]{225{,}49562} = 6{,}0866 +$ feet, the length of the side required.

PROBLEM X.

EXAMPLES.

1. $3 \times 3 \times 3 \times 2{,}18169 = 58{,}90563$ cubic feet, the solidity required.

2. $4 \times 4 \times 4 \times 2{,}18169 = 139{,}62816$ cubic feet, the solidity required.

3. $6 \times 6 \times 6 \times 2{,}18169 = 471{,}245 +$ cubic feet, the solidity required.

PROBLEM XI.

EXAMPLES.

1. $24 \div 2{,}18169 = 11 +$ the cube of the side, and $\sqrt[3]{11} = 2{,}22398 +$ feet, the length of the side.

2. $128 \div 2{,}18169 = 58{,}67$ the cube of the side; then $\sqrt[3]{58{,}67} = 3{,}841 +$ feet, the length of the side.

3. $16 \div 2{,}18169 = 7{,}333764 +$ the cube of the side; then $\sqrt[3]{7{,}333764} = 1{,}9428 +$ feet, the length of the side required.

## SECTION XI.

### OF REGULAR BODIES.

#### PROBLEM I.

EXAMPLES.

1. $8 \times 8 \times 8 \times ,5236 \div ,11785 = 2274,7832$ — the cube of the side; then $\sqrt[3]{2274,7832} = 13,1516 +$ inches, the side required.

2. $6 \times 6 \times 6 \times ,5236 \div 11785 = 959,674162$ the cube of the side; then $\sqrt[3]{959,674162} = 9,8637 +$ feet, the side required.

#### PROBLEM II.

EXAMPLES.

1. $20 \times 20 \times 20 \times ,4714 \div ,11785 = 32000$ the cube of the side; then $\sqrt[3]{32000} = 31,748 +$ inches, the length of the side required.

2. $3 \times 3 \times 3 \times ,4714 \div ,11785 = 108$ the cube of the side; then $\sqrt[3]{108} = 4,7622 +$ feet, the length of the side required.

#### PROBLEM III.

EXAMPLES.

1. $4 \times 4 \times 4 \times 7,66312 \div ,4714 = 1040,39$ — the cube of the required side; then $\sqrt[3]{1040,39} = 10,1328$ feet, the length of the side.

2. $6 \times 6 \times 6 \times 7,66312 = 3511,315$ the cube of the side; then $\sqrt[3]{3511,315} = 15,2$ — feet, the length of the required side.

#### PROBLEM IV.

EXAMPLES.

1. $12 \times 12 \times 12 \times 2,18169 \div 7,66312 = 491,96154$ + the cube of the side; then $\sqrt[3]{491,96154} = 7,89 +$ feet, the side required.

2. $10 \times 10 \times 10 \times 2,18169 \div 7,66312 = 287,30987$ — the cube of the side; then $\sqrt[3]{287,30987} = 6,5985 +$ feet, the side required.

# SECTION XII.

## MENSURATION OF RINGS.

### PROBLEM I.

EXAMPLES.

1. $12 + 3 = 15$ the sum of the inner diameter and thickness of the ring; then $15 \times 3 \times 9{,}8696 = 444{,}132$ square inches, the area required.

2. $18 + 4 = 22$ inches the sum of the inner diameter and thickness of the ring; then $22 \times 4 \times 9{,}8696 = 868{,}5248$ inches, the area of the surface required.

3. $18 + 2 = 20$, and $20 \times 2 \times 9{,}8696 = 394{,}784$ square inches.

### PROBLEM II.

EXAMPLES.

1. $9 + 3 = 12$ the sum of the inner diameter and thickness of the ring, and $3 \div 2 = 1{,}5$, half the thickness of the ring; then $1{,}5 \times 1{,}5 \times 12 \times 9{,}8696 = 266{,}4792$ cubic inches, the solidity required.

2. $12 + 4 = 16$ the sum of the inner diameter and thickness of the ring, and $4 \div 2 = 2$ inches the semidiameter of the ring; then $2 \times 2 \times 16 \times 9{,}8696 = 631{,}6544$ cubic inches, the solidity required.

3. $16 + 2 = 18$ the sum of the inner diameter and diameter of the ring, and $2 \div 2 = 1$ the semidiameter of the ring, whose square will also be 1; $1 \times 18 \times 9{,}8696 = 197{,}6528$ cubic inches, the solidity required.

4. $12 + 5 = 17$ inches the sum of the given diameters, and $5 \div 2 = 2{,}5$ inches half the diameter of the ring; then $2{,}5 \times 2{,}5 \times 17 \times 9{,}8696 = 1048{,}645$ cubic inches, the solidity.

### PROBLEM III.

EXAMPLES.

1. $789{,}568 \div 9{,}8696 = 80$, and $80 \div 4 = 20$, and $20 - 4 = 16$ inches, the diameter required.

2. 138,1744 ÷ 9,8696 = 14, and 14 ÷ 1 = 14 the sum of the inner diameter and that of the ring; then 14 — 2 = 12 inches, the inner diameter of the ring.

3. 1728 ÷ 9,8696 = 175,08 +; then 175,08 + ÷ 4 = 43,77 + the sum of the given and required diameters; then 43,77 — 4 = 39,77 inches, the inner diameter required.

4. 1728 × 4 ÷ 9,8696 = 700,3323 +, and 3,5 ÷ 2 = 1,75; then 1,75 × 1,75 = 3,0625; then 700,3323 ÷ 3,0625 = 228,68 inches, the sum of the given and required diameters; then 228,68 — 3,5 = 225,18 inches the inner diameter = 18,765 feet, the inner diameter required.

5. 1 ÷ 9,8696 = ,10132123, and $\frac{1}{8}$ ÷ 2 = $\frac{1}{16}$, half the thickness of the ring; then $\frac{1}{16}$ × = $\frac{1}{256}$, the square of half the thickness of the ring; then ,10132123 ÷ $\frac{1}{256}$ = 25,9382 + inches the sum of the given and required diameters, and 1 ÷ 8 = ,125 = to $\frac{1}{8}$; then 25,9382 — ,125 = 25,8132 inches, the inner diameter required.

## SECTION XIII.

### OF MEASURING CASKS.

#### PROBLEM I.

EXAMPLES.

1. $24 \times 24 = 576$ the head diameter, and $32 \times 32 \times 2 = 2048$ twice the square of the diameter at the bung; then $2048 + 576 = 2624$ their sum; then $2624 \times 40 \div 882 = 119$ wine gallons the solidity required.

2. $25 \times 25 = 625$ the square of the head diameter, and $34 \times 34 \times 2 = 2312$ twice the square of the bung diameter; then $2312 + 625 = 2937$ the sum; then $2937 \times 45 \div 1077 = 122{,}716$ — ale gallons the solidity required.

3. $23 \times 23 = 529$ the square of the head diameter, and $32 \times 32 \times 2 = 2048$ twice the square of the bung diameter; then $2048 + 529 = 2577$ the sum; then $2577 \times 42 \times \div 882 = 122{,}7147$ — wine gallons the solidity required.

4. $32 \times 32 = 1024$ the square of the head diameter, and $40 \times 40 \times 2 = 3200$ twice the square of the bung diameter; then $3200 + 1024 = 4224$ the sum; then $4224 \times 50 \div 1077 = 196{,}1$ ale gallons the solidity required.

#### PROBLEM II.

EXAMPLES.

1. $24 \times 24 = 576$ the square of the head diameter, and $31 \times 31 \times 2 = 1922$ twice the square of the bung diameter, and $1922 + 576 = 2498$ the sum, and $31 - 24 = 7$ the difference of the diameters, and $7 \times 7 = 49$ the square of the difference; then $49 \times {,}4 = 19{,}6$; then $2498 - 19{,}6 = 2478{,}4$; then $2478{,}4 \times 40 \times \div 882 = 112{,}4$ — wine gallons the content required.

2. $24 \times 24 = 576$ the square of the head diameter,

and 32 × 32 × 2 = 2048 twice the square of the bung diameter, and 2048 + 576 = 2624 the sum, and 32 — 24 = 8 inches the difference of the two diameters, and 8 × 8 = 64 the square of the difference; then 64 × ,4 = 25,6, and 2624 — 25,6 = 2598,4; then 2598,4 × 42 ÷ 1077 = 101,33 ale gallons, the content required.

3. 25 × 25 = 625 the square of the head diameter, and 32 × 32 × 2 = 2048 twice the square of the bung diameter, and 2048 + 625 = 2673 the sum, and 32 — 25 = 7, and 7 × 7 = 49 the square of the difference of the diameters; then 49 × ,4 = 19,6 and 2673 — 19,6 = 2653,4 and 2653,4 × 45 ÷ 882 = 135,3775 + wine gallons, the content required.

### PROBLEM III.

#### EXAMPLES.

1. 24 × 24 = 576 the square of the head diameter, and 32 × 32 = 1024 the square of the bung diameter, and 1024 + 576 = 1600 the sum of the squares of both diameters; then 1600 × 40 × ,0017 = 108,8 wine gallons, the content required.

2. 20 × 20 = 400 the square of the head diameter, and 30 × 30 = 900 the square of the bung diameter; then 900 + 400 = 1300 the sum of the squares of both diameters, and 1300 × 50 × 0014 = 91 ale gallons, the content required.

3. 20 × 20 = 400 the square of the head diameter, and 30 × 30 = 900 the square of the bung diameter; then 900 + 400 = 1300 the sum of their squares, and 1300 × 60 × ,0017 = 132,6 wine gallons, the content required.

### PROBLEM IV.

#### EXAMPLES.

1. 24 × 32 = 768 the product of the head and bung diameter, and 32 — 24 = 8 the difference of the two diameters, and 8 × 8 ÷ 3 = 21,333 +, one-third of the square of the difference; then 768 + 21,333 = 789,333; then 789,333 × 40 × ,7854 ÷ 231 = 107,3494 — wine gallons, the content required.

2. $21 \times 30 = 630$ the product of the two diameters, and $30 - 21 = 9$ inches the difference between the two diameters, and $9 \times 9 \div 3 = 27$, one-third of the square of the difference of the two diameters; then $630 + 27 = 657$ the sum; then $657 \times 50 \times ,7854 \div 282 = 91,49 +$ ale gallons, the solidity required.

3. $18 \times 30 = 540$ the product of the two diameters, and $30 - 18 = 12$ inches the difference of the two diameters, and $12 \times 12 \div 3 = 48$, one-third of the difference of the squares of the two diameters; then $540 + 48 = 588$, and $588 \times 50 \times ,7854 \div 231 = 99,96$ wine gallons, the solidity required.

## PROBLEM V.

### EXAMPLES.

1. $32 \times 32 + 24 \times 24 = 1600$ the sum of the squares of the head and bung diameters, and $28,75 \times 2 = 57,5$, double the diameter of the middle between the bung and head diameters, and $57,5 \times 57,5 = 3306,25$ its square; then $3306,25 + 1600 = 4906,25$; then $4906,25 \times 40 \times ,0005\frac{2}{3} = 111,2$ wine gallons, the content required.

2. $30 \times 30 + 20 \times 20 = 1300$ the sum of the squares of the head and bung diameters, and $26 \times 2 = 52$, twice the middle diameter between the bung and head, and $52 \times 52 = 2704$; then $2704 + 1300 = 4004$, and $4004 \times 50 \times ,00046666 = 93,425 +$ ale gallons, the solidity required.

3. $12 \times 12 + 16 \times 16 = 500$ the sum of the squares of the head and bung diameters, and $14,5 \times 2 = 29$ inches twice the diameter in the middle between the head and bung diameters, and $29 \times 29 = 841$ its square; then $500 + 841 = 1341$ the sum of the squares, and $1341 \times 20 \times ,00056666 = 15,197 +$ wine gallons, the solidity required.

## PROBLEM VI.

### EXAMPLES.

1. $32 \times 32 \times 39 = 39936$, thirty-nine times the square of the bung diameter, and $24 \times 24 \times 25 = 14400$, twen-

ty-five times the head diameter, and $24 \times 32 \times 26 = 19968$, twenty-six times the product of the two diameters, and $39936 + 14400 + 19968 = 74304$, and $74304 \times 40 \times ,00034 \div 9 = 112,2816$ wine gallons, the solidity required.

2. $24 \times 24 \times 25 = 14400$, twenty-five times the square of the head diameter, and $36 \times 36 \times 39 = 50544$, thirty-nine times the square of the bung diameter, and $24 \times 36 \times 26 = 22464$, twenty-six times the product of the head and bung diameters; then $22464 + 50544 + 14400 = 87408$, and $87408 \times 50 \times ,00034 \div 11 = 135,085$ ale gallons, the solidity required.

3. $30 \times 30 \times 39 = 35100$, thirty-nine times the square of the bung diameter; $20 \times 20 \times 25 = 10000$, twenty-five times the square of the head diameter; $20 \times 30 \times 26 = 15600$, twenty-six times the product of the head and bung diameters, and $35100 + 10000 + 15600 = 60700$, and $60700 \times 48 \times ,00034 \div 9 = 110,0693 +$ wine gallons, the capacity of the cask required.

## PROBLEM VII.

### EXAMPLES.

1. $32 \times 32 = 1024$ the square of the diameter of the cask at the surface of the liquor, and $24 \times 24 = 576$ the square of the diameter of the nearest end, and $29 \times 2 = 58$, twice the diameter in the middle between the diameters; then $58 \times 58 = 3364$, and $3364 + 1024 + 576 = 4964$ the sum of those squares, and $12 \div 6 = 2$ inches, one-sixth of the depth of the liquor; then $4964 \times 2 \times ,0034 = 33,75 +$ wine gallons, the quantity required.

2. $24 \times 24 = 576$ the square of the head diameter, and $29 \times 29 = 841$ the square of the diameter at the surface of the liquor, and $27 \times 2 = 54$ inches, twice the diameter in the middle between the other two, and $54 \times 54 = 2916$ its square; then $2916 + 841 + 576 = 4333$, and $4333 \times ,0028 + 10 \div 6 = 20,22 +$ ale gallons, the quantity required.

### PROBLEM VIII.

#### EXAMPLES.

1. 8 ÷ 32 = ,25 the tabular versed sine, and ,153546 its corresponding area; then ,153546 × 92 × 1,25 = 17,65779 ale gallons, the ullage required.

2. 24)6 = ,25 the tabular versed sine, and ,153546 the corresponding area; then ,153546 × $49\frac{2}{3}$ × 1,25 = 9,5326 + wine gallons it lacks of being full; then 49,6666 — 9,5326 = 40,134 wine gallons the quantity left in the cask, and consequently the ullage required.

3. 12 ÷ 32 = ,375 the tabular versed sine, and ,269013 the corresponding area; then ,269013 × 119 × 1,25 = 40 + wine gallons the quantity lacking to fill the cask; then 119 — 40 = 79 wine gallons, the cask contains.

## SECTION XIV.

### ARTIFICERS' WORK.

#### PROBLEM I.

EXAMPLES.

1. $48 \times 12{,}5 \div 272{,}25 = 2{,}20385$; then $2{,}20385 \times 5 \div 3 = 3{,}673 +$ square perches, the answer required.

2. $63{,}25 \times 16{,}75 \div 272{,}25 = 3{,}8914$ square perches $2\frac{1}{2}$ bricks thick; then $3{,}8914 \times 5 \div 3 = 6{,}452 +$ square perches, the number sought.

3. $60 \times 2 = 120$ feet the length of the two side walls, and $35{,}5 \times 2 = 71$ feet the length of the two ends; then $120 + 71 = 191$ feet the whole length of the wall, and $191 \times 30 = 5730$ square feet, and $35{,}5 \times 10 = 355$ square feet in both gables; then $5730 + 355 = 6085$ square feet in the whole wall; $6085 \div 272{,}25 = 22{,}35$ + perches two bricks in thickness; then $22{,}35 \times 4 \div 3 = 29{,}8$ square perches, the answer required.

#### PROBLEM II.

EXAMPLES.

1. $45{,}5 \times 2 = 91$ feet the length of the sides, and $24 \times 2 = 48$ feet the length of the two ends; then $91 + 48 = 139$ feet the whole length of the wall; then $139 \times 6{,}75 \times 2 \div 16{,}5 = 113{,}72 +$ perches the number required.

2. $42 \times 2 = 84$ feet the length of the two sides, and $26 - 4 = 22$ feet the breadth in the inner side; then $22 \times 2 = 44$ feet the length of the two ends; then $84 + 44 = 128$ feet the whole length of the wall; then $128 \times 6{,}5 \times 2 \div 16{,}5 = 100{,}8484 +$ perches, the content of the cellar wall; then $100{,}8484 \times {,}40 = \$40{,}339$ the expense required.

3. $36 \times 2 = 72$ feet the length of the two sides, and $24 \times 2 = 48$ feet the length of the two ends; then 72

$+ 48 = 120$ feet the whole length of the wall; then $120 \times 6 \times 1{,}75 \div 16{,}5 = 76{,}36$ perches the content of the wall; then $76{,}36 \times {,}90 = \$68{,}724$ the expense required.

4. $58 \times 2 = 116$ feet the length of the two sides, and $15 \times 2 = 30$ inches or 2,5 feet twice the thickness of the wall; then $26 - 2{,}5 = 23{,}5$ length of each end at the inner side; then $23{,}5 \times 2 = 47$ feet the length of the two ends; then $116 + 47 = 163$ feet the whole length of the wall; and $163 \times 22 = 3586$ feet the solidity of the upright part, 15 inches thick; then $23{,}5 \times 12 = 282$ feet the solidity of the gables, 15 inches thick; then $3586 + 282 = 3868$; then $3868 \times 1{,}25 \div 16{,}5 = 293$ + perches, without deductions for doors and windows, and $293 \times {,}56 = \$164{,}08$ the expense of workmanship, and $8 \times 4{,}5 \times 2 = 72$, and $28 \times 3{,}5 \times 6 = 588$, and $588 + 72 = 660$ feet 15 inches thick the deductions for doors and windows; then $660 \times 1{,}25 \div 16{,}5 = 50$ perches; then $293 - 50 = 243$ perches after the deductions are made, and $243 \times {,}44 = \$106{,}92$, the value of the stone.

### PROBLEM III.

#### EXAMPLES.

1. $60 \times 28{,}75 \div 100 = 17{,}25$ squares, the number required.

2. $45{,}5 \times 26{,}75 \times 3 \times 1{,}36 \div 100 = \$49{,}6587$, the expense required.

3. $96{,}75 \times 11{,}5 \div 100 = 11{,}12625$ squares, the answer required.

4. 30,25 feet the breadth of the building; $\frac{3}{5} = {,}6$; then $30{,}25 \times 6 = 18{,}15$ feet the length of the rafter; then $52{,}5 \times 18{,}15 \times 2 \div 100 = 19{,}0575$ the number of squares; then $19{,}0575 \times 1{,}25 = \$23{,}82$, the expense of roofing.

### PROBLEM IV.

#### EXAMPLES.

1. $48{,}5 \times 36{,}25 = 1708{,}125$ square feet, the content.

2. 26,75 the breadth of the building; then $26{,}75 \times$

,75 = 20,0625 the length of the rafter; then 20,0625 × 2 = 40,125 the girt, and 9 inches ,75 of a foot; then ,75 × 2 = 1,5 feet the whole projection; then 40,125 + 1,5 = 41,625 the whole girt including the projection; then 41,625 × 42,5 × 3,4 ÷ 100 = $60,148 + the expense required.

PROBLEM V.

EXAMPLES.

1. 64,75 × 24,5 ÷ 9 = 32,375 square yards, the content required.

2. 150 × 11,5 ÷ 9 = 191,67 — square yards.

3. 21,6666 × 2 = 43,3332 feet the length of the two sides, and 17,3333 × 2 = 34,6666 feet the length of the two ends; then 43,3332 + 34,6666 = 78 feet the whole length around the room; then 78 × 10,25 = 799,5 square feet; then 21,6666 × 17,3333 = 375,5 feet the area over head; then 799,5 + 375,5 = 1175 feet, and 7 × 3,25 = 22,75 the area of the door; then 1175 — 22,75 ÷ 9 = 128,028 — square yards of plastering; then 128,028 × ,07 = $8,961, the expense required.

PROBLEM VI.

EXAMPLES.

1. 45 × 18,75 × ,40 ÷ 9 = $37,5 the expense required.

2. 64,75 × 45,5 × ,45 ÷ 9 = $147,30, the expense required.

3. 1,20 × 4 ÷ 40 = 12 yards, the side of the square required.

PROBLEM VII.

EXAMPLES.

1. 40 × 40 × ,7854 ÷ 2 = 628, 32 square feet the area of the end; then 628,32 × 124 = 77911,68, the solidity required.

2. 40 × 80 × 12 × ,7854 = 30159,36, the solidity required.

3. 48 ÷ 2 = 24, half the chord, and 24 × 24 × 7 =

4032, seven times the square of half the chord, and 18 $\times$ 18 $\times$ 2 = 648, twice the square of the versed sine; then 4032 + 648 = 4680, and 4680 $\times$ 7 = 32760, and $\sqrt{32760}$ = 181 —, and 24 $\times$ 24 + 18 $\times$ 18 = 900, and $\sqrt{900}$ = 30; then 30 $\times$ $\frac{4}{3}$ = 40, and 181 + 40 = 221; then 221 $\times$ 18 $\times$ ,16 $\times$ 2 = 1272,96 the area of the two circular segments, and 48 $\times$ 48 $\times$ ,433013 = 997,661952 the area of the triangular part; then 1272,96 + 997-,661952 = 2270,605752 the area of the end; then 2270-,60572 $\times$ 60 = 136236,34512, the solidity required.

### PROBLEM VIII.

#### EXAMPLES.

1. 3,1416 $\times$ 20 $\times$ 120 = 7539,84 square feet, the area of the concave surface required.

2. 3,416 $\times$ 15 $\times$ 90 = 1413,72 square feet, the area of the concave surface required.

### PROBLEM IX.

#### EXAMPLES.

1. 60 $\times$ 60 $\times$ ,7854 = 2827,44 square feet the area of the base and 60 $\div$ 2 = 30 feet the height; then 30 $\times$ $\frac{2}{3}$ = 20 feet, two-thirds the height; then 2827,44 $\times$ 20 = 56548,8 cubic feet, the solidity of the dome required.

2. 90 $\times$ 90 $\times$ 3,1416 = 25446,96 square feet the area of the base, and 90 $\div$ 2 = 45 feet the height of the dome; then 45 $\times$ $\frac{2}{3}$ = 30 feet, two-thirds of the height, and 25446,96 $\times$ 30 = 763408,8 cubic feet, the solidity required.

### PROBLEM X.

#### EXAMPLES.

1. 20 $\times$ 20 $\times$ 4,828427 $\times$ 2 = 3862,7416 square feet the area of its surface; then 3862,7416 $\div$ 9 $\times$ ,12 = \$51,503 +, the expense required.

2. 20 $\times$ 20 $\times$ 2,598076 $\times$ 2 $\div$ 9 = 230,94 + square yards the area of the surface; then 230,94 $\times$ ,15 = \$34-,641, the expense required.

PROBLEM XI.

EXAMPLES.

1. Here $16 - 2 \times 2 = 12$; then the flat part of the ceiling is 16 feet by 12; then $16 \times 2 + 12 \times 2 = 56$ feet the perimeter; then $56 \div 4 = 14$ feet, one-fourth of the perimeter; then $14 \times 2 = 28$; then multiplying by the projection 2 feet $= 56$; then $3{,}1416 \times 56 = 175{,}9296 = A$; then $20 - 16 = 4$; then $4 \times 4 = 16$; then $2 \times \frac{2}{3} \times 16 = 21{,}333 + = B$; $16 \times 12 = 192$ square feet the area of the flat ceiling; then $192 \times 2 = 384$ the area of the flat ceiling multiplied by the height of the arch; then $384 + 175{,}9296 + 21{,}333 = 581{,}2629 +$ cubic feet, the solidity required.

2. $40 \times 40 \times 25 \times {,}7854 = 31416$ cubic feet, the solidity of the upright part, and $40 \times 40 \times {,}7854 \times 5 \times \frac{2}{3} = 4188{,}8$ the solidity of the arch; then $31416 + 4188{,}8 = 35614{,}8$ cubic feet, the solidity required.

PROBLEM XII.

EXAMPLES.

1. $12 \times 12 \times 6 \times {,}904 = 781{,}056$ cubic feet, the solidity required.

2. $20 \times 20 \times 6 \times {,}904 = 2169{,}6$ cubic feet, the solidity required.

PROBLEM XIII.

EXAMPLES.

1. $16 \times 16 \times 1{,}1416 = 292{,}2496$ square feet, the area required.

2. $14 \times 14 \times 1{,}1416 = 223{,}7536$ square feet, the area required.

## SECTION XV.

### MENSURATION OF SOLIDS.

#### PROBLEM I.

EXAMPLES.

1. $8 + 2 = 10$, and $10 \times 9 \times 8 \div 6 = 120$, the number of shot required.

2. $30 + 2 = 32$; then $32 \times 31 \times 30 \div 6 = 4960$, the number of balls required.

#### PROBLEM II.

EXAMPLES.

1. $20 \times 2 + 1 = 41$, and $20 + 1 = 21$; then $21 \times 41 \times 20 \div 6 = 2870$, the number required.

2. $30 \times 2 + 1 = 61$, and $30 + 1 = 31$; then $61 \times 31 \times 30 \div 6 = 9455$, the number of shot required.

3. $12 \times 2 + 1 = 25$, and $12 + 1 = 13$; then $25 \times 13 \times 12 \div 6 = 650$ shot, the number required.

#### PROBLEM III.

EXAMPLES.

1. $30 \times 2 + 1 = 61$, and $31 - 1 = 30$, and $30 \times 3 = 90$, and $90 + 61 = 151$; then $151 \times 31 \times 30 \div 6 = 23405$, the number of shells required.

2. $20 \times 2 + 1 = 41$, and $24 - 1 = 23$, and $23 \times 3 = 69$, and $69 + 41 = 110$; then $110 \times 21 \times 20 \div 6 = 7700$, the number required.

3. $30 \times 2 + 1 = 61$, and $41 - 1 = 40$, and $40 \times 3 = 120$; then $120 + 61 = 181$; then $181 \times 31 \times 30 \div 6 = 28055$ shot, the number required.

#### PROBLEM IV.

EXAMPLES.

1. $40 + 2 = 42 \times 41 \times 40 \div 6 = 11480$ the number if the pile was complete, and $19 + 2 = 21$, and $21 \times 20 \times$

$19 \div 6 = 1330$; then $11480 - 1330 = 10150$, the number of shot in the unfinished pile.

2. $24 + 2 = 26$, and $26 \times 25 \times 24 \div 6 = 2600$ the number of shot there would be if the pile was complete, and $7 + 2 = 9$, and $9 \times 8 \times 7 \div 6 = 84$ the number it would take to complete the pile; then $2600 - 84 = 2516$, the number required.

3. $24 \times 2 + 1 = 49$, and $24 + 1 = 25$; then $49 \times 25 \times 24 \div 6 = 4900$ the number there would be if the pile were complete, and $7 \times 2 + 1 = 15$; then $15 \times 8 \times 7 \div 6 = 140$ the number it would take to complete it; then $4900 - 140 = 4760$, the number required.

4. $30 \times 2 + 1 = 61$, and $30 + 1 = 31$; then $61 \times 31 \times 30 \div 6 = 9455$ the number if the pile were complete; then $11 \times 2 + 1 = 23$, and $11 + 1 = 12$; then $23 \times 12 \times 11 \div 6 = 506$ the number it would take to finish the pile; then $9455 - 506 = 8949$, the number of shot contained in the unfinished pile.

5. $40 \times 20 = 800$ the number contained in the lower tier; $40 - 12 = 28$, and $20 - 12 = 8$; then $28 \times 8 = 224$ the number in the upper course; then $800 + 224 \times 12 \div 2 = 6144$, the number required.

6. $30 \times 20 = 600$ the number in the lower course, and $30 - 8 = 22$, and $20 - 8 = 12$; then $22 \times 12 = 264$ the number in the upper course, and $600 + 264 = 864$ the sum of the two extremes; then $864 \times 8 \div 2 = 3456$, the number of shot required.

## SECTION XVI.

### OF SPECIFIC GRAVITY.

#### PROBLEM I.

EXAMPLES.

1. 10 — 7 = 3 *lbs.* the weight lost in water; then 3 : 10 :: 1000 : 3333 avoirdupois ounces the specific gravity, or the weight of one cubic foot of the stone.

2. 24 — 20 = 4 *lbs.* the weight lost in water; then 4 : 1000 :: 24 : 6000, the specific gravity required.

#### PROBLEM II.

EXAMPLES.

1. 18 + 15 = 33 the weight of the compound in air, and 18 — 16 = 2 *lbs.* the weight of the copper lost in water, and 33 — 6 = 27 the weight of the compound lost in water; then 27 — 2 = 25 the actual weight of the compound lost in water; then 25 : 1000 :: 15 : 600, the specific gravity required.

2. 37 *lbs* + 18 = 55 the weight of the compound in air, and 18 — 16 = 2 *lbs.* the weight of the copper lost in water, and 55 — 13 = 42, and 42 — 2 = 40 *lbs.* the weight of the compound lost in water; then 40 : 1000 :: 37 : 925 ounces, the specific gravity required.

#### PROBLEM III.

EXAMPLES.

1. 40 — 34,61 = 5,39 the weight lost in the fluid; then 40 : 5,39 :: 7425 : 1000, the specific gravity of the fluid being water.

2. 40 — 35,375 = 4,625 the weight lost by weighing in the fluid; then 40 : 8000 :: 4,625 : 925, the specific gravity of the fluid = to proof spirits.

PROBLEM IV.

EXAMPLES.

1. 9000 the specific gravity of copper, and 7320 the specific gravity of tin; then 9000 — 7320 = 1680, and 8784 the specific gravity of the compound; then 1680 × 8784 = 14757120, and 9000 — 8784 = 216; then 216 × 7320 = 1581120, and 8784 — 7320 = 1464, and 1464 × 9000 = 13176000; then 14757120 : 112 : : 13176000 : 100 *lbs.* weight of copper, and 112 — 100 = 12 *lbs.* tin.

2. 9000 — 7320 = 1680; then 1680 × 8440 = 14179200, and 9000 — 8440 = 360; then 360 × 7320 = 4099200, and 8440 — 7320 = 1120, and 1120 × 9000 = 10080000; then 14179200 : 48 : : 10080000 : 34,1225 + *lbs.* the weight of copper, and 48 — 34,1225 + = 13,8775 *lbs.* of tin.

PROBLEM V.

EXAMPLES.

1. 2700 : 1728 : : 112 : 71,68 cubic inches, the solidity required.

2. 3000 : 1728 : : 300 : 172,8 cubic inches, the solidity required.

PROBLEM VI.

EXAMPLES.

1. 2,5 × ,5 × 16 × 12 = 240 cubic inches the solidity of the bar; then 1728 : 7645 : : 240 : 1061,8 avoirdupois ounces = to 66,3625 pounds.

2. 1 *lb.* : 2700 : : 63 × 12 × 12 : 24494400 avoirdupois ounces, which divided by 16 = 1530900 pounds; then 1530900 ÷ 2240 = 683$\frac{7}{16}$ tons, the weight required.

3. 282 solid inches make a gallon ale measure, and 8 pints make a gallon; then 282 — 8 = 35,25 cubic inches the solidity of the gunpowder; then 1728 : 922 : : 35,25 : 18,8 ounces, the weight required.

4. 1 : 755 : : 128 : 96640 ounces; then 96640 ÷ 16 = 6040 pounds, which divided by 112 = 54 — hundred weight nearly.

5. $12 \times 12 \times 16 \times 14 = 32256$ cubic inches the solidity of the stick of timber; then 1728 : 925 :: 32256 : 17266 + ounces; then $17266 \div 16 = 1079 +$ *lbs.* the weight required.

6. $4 \times 2{,}5 \times 1{,}75 = 17{,}5$ cubic feet, the solidity of the stone; then 1 : 2520 :: 17,5 : 44100 ounces, the weight of the stone = 2756,25 *lbs.*

7. $\frac{3}{4} = {,}75$; then $,75 \times ,75 \times ,75 \times ,5236 = ,22089375$ cubic inches, the solidity of the ball; then 1728 : 11325 :: ,22089375 : 1,4477 ounces, the weight of the ball required.

8. 1728 : 17724 :: 1 : 10,257 — avoirdupois ounces, the weight required.

## SECTION XVII.

### MISCELLANEOUS QUESTIONS.

1. $16,75 \times 16,75 = 280,5625$ square feet, the number required.

2. $24 \times 16,75 \times 1,50 \div 100 =$ \$6,03, the expense required.

3. $35 \times 40 \div 10 = 140$ acres, the area required.

4. $24 \div 2 = 12$ chains half the perpendicular; then $36 \times 12 \div 10 = 31,2$ acres the area, and $36 \times 36 + 24 \times 24 = 1872$ the square of the hypotenuse; then $\sqrt{1872} = 43,2666 +$ chains, the length of the hypotenuse.

5. $4,5 \times 10 = 45$ square chains the area, and $4 \div 2 = 2$; then $2 \times 2 + 45 = 49$ the square of the half sum of the two sides; then $\sqrt{49} = 7$ chains the half sum; then $7 + 2 = 9$ chains, the longer side, and $7 - 2 = 5$ chains, the shorter.

6. $144 \div 8 = 18$ inches, the length required.

7. $3,1416 \times 25 = 78,54$ rods, the length of the wall required.

8. $160 \times 12,5664 = 2010,624$ the square of the circumference, then $\sqrt{2010,624} = 44,84 -$ rods, the length of the wall required.

9. $30,25 \times 80 \times 9 = 21708$ square feet the area of the garden; $21780 \div 7854 = 27731,0924$ the square of the diameter of the garden; then $\sqrt{27731,0924} = 166,52$ feet the diameter of the garden; then $166,52 + 4 = 170,52$ feet the diameter including the wall; then $170,52 \times 170,52 \times ,7854 = 22837,131$ square feet the area of the garden including the wall; then $22837,131 - 21780 = 1057,131$ square feet of land the wall stands on; then $1057,131 \times 4,5 \div 16,5 = 288,3$ perches, the answer required.

10. $160 \times 30{,}25 \times 9 \div 12 = 3630$, the number of men required.

11. $46 \times 36 \times {,}7854 \div 160 = 8{,}1289$ acres, the area required.

12. $4 \times 12 = 48$ inches the length of the pole; then $48 \div 2 = 24$ inches to the centre; then $24 + 6 = 30$ inches the distance from one end to the place where the weight is suspended, and $48 - 30 = 18$ inches from the other end; then $48 : 128 :: 30 : 80$ the number of pounds carried by one, and $128 - 80 = 48$ pounds carried by the other; then $80 - 48 = 32$ *lb.* the difference required.

13. $18 \times 18 \times 18 \times {,}5236 \div 1728 = 1{,}76715$ cubic feet, the solidity required.

14. $60 \times 60 \times {,}7854 = 2827{,}44$ square chains the area of the circle; then $2827{,}44 \div 3 = 942{,}48$ square chains to be cut off; then $942{,}48 \div 60 \times 60 = {,}26180$ the tabular area, the corresponding versed sine is ,3675352 which multiplied by 60 the given diameter $= 22{,}052 +$ chains the versed sine of the segment; then $60 - 22{,}052 = 37{,}948$ and $37{,}948 \times 22{,}052 = 836{,}829296$ the square of half the chord; then $\sqrt{836{,}829296} = 28{,}928 -$ chains, half the chord; then $28{,}928 \times 2 = 57{,}856$ chains, the length of the chord required.

15. $64 \div 2 = 32$ half the chord; then $32 \times 32 \div 16 = 64$, and $64 + 16 = 80$ feet, the diameter of the circle required.

16. $10 \times 10 = 100$ square chains the area; then $100 \times 5 \div 4 = 125$ the square of the longer side; then $\sqrt{125} = 11{,}18 +$ chains the length of the longer side; then $11{,}18 \times 4 \div 5 = 8{,}944$ chains the shorter side.

17. $30{,}25 \times 40 = 1210$ square yards, the area; then $1210 \div {,}7854 = 1540{,}6162 +$ the square of the diameter of the orangery; then $\sqrt{1540{,}6162} = 39{,}25 +$ yards the diameter; then $39{,}25 \div 2 = 19{,}625$ yards the length of the line required.

18. $17 \times 17 \times {,}7854 \times 42 \div 144 = 66{,}2 +$ cubic feet the solidity required.

19. $6 \times 6 \times 6 \times {,}5236 = 113{,}0976$ cubic inches the solidity of the globe; then $\frac{1}{20} \times \frac{1}{20} = \frac{1}{400}$; then $\frac{1}{400} \times {,}7854$

$=,0019635$ the area of the end of the wire; then $113,0976 \div ,0019635 = 57600$ inches $= 1600$ yards, the length of the wire required.

20. The specific gravity of cast iron is 7425; then $7425 : 1728 :: 400 \times 16 : 1489,45 +$ cubic inches, the solidity required.

21. $40 \times 40 \times ,7854 \times 3 \div 144 = 26,18$ cubic feet, the solidity required.

22. $48 \times 27 \times 6 \div 27 = 288$ solid yards, the solidity required.

23. 4 feet 6 inches $= 54$ inches; then $54 \times 54 \times 54 \div 2150,4252 = 73,2245 +$ bushels the solidity required.

24. $8 \times 8 \times 8 = 512$ the cube of the altitude of the cone; then $512 \times 7 \div 8 = 448$ the cube of the altitude left at the vertex after the section is taken off; then $\sqrt[3]{448} = 7,65 +$ feet, the altitude required.

25. $40 \div 2 = 20$ half the length of the perpendicular; then $50 \times 20 = 1000$ square chains the arc of the triangle; then $1000 : 50 \times 50 :: 1000 \div 2 : 1250$ the square of the base left at the verticle angle after the division; then $\sqrt{1250} = 35,3553 +$ chains the length of the longer base, and $50 - 35,3553 = 14,6447$ chains the shorter.

26. $10 \times 4 + 3 = 43$ roods, and $43 \times 40 + 8 = 1728$ square rods the area of the parallelogram; then $1728 \times 2 = 3456$ twice the area, and $60 \times 60 - 3456 = 144$ the square of the difference between the length and breadth; then $\sqrt{144} = 12$ rods the difference, and $12 \div 2 = 6$ rods half the difference, and $60 \times 60 + 3456 = 7056$ the square of the sum of the two sides; then $\sqrt{7056} = 84$ rods the sum of the two sides; then $84 \div 2 = 42$ the half sum and $42 + 6 = 48$ rods the longer side, and $42 - 6 = 36$ the shorter, hence the two sides are 48 and 36 rods.

27. $6 \times 6 \times 1,5 \times ,7854 = 42,4116$ cubic inches contained in the plank; then $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$ the solidity of one of the pieces, and $42,4116 \div \frac{1}{64} = 2714 +$ pieces.

28. $5 \times 12 = 60$ inches, and $6 \times 12 = 72$, and $7 \times 12 = 84$ inches; then $72 \times 60 = 4320$, and $72 - 60 =$

12 inches the difference between the top and bottom diameters; then $12 \times 12 \div 3 = 48$, one-third of the square of the difference; then $4320 + 48 = 4368$; then $4368 \times 84 \times ,7854 \div 282 = 1021,8889$ ale gallons, the content required.

29. $7 \times 7 = 49$ the square of the diameter, and $2,5 \times 2,5 = 6,25$ the square of the diameter of the hollow space; then $49 - 6,25 = 42,75$; then $42,75 \times 30 \times ,7854 \div 16,5 = 61,047$ perches, the quantity of stone required.

30. $4 \times 3 \times ,7854 = 9,4248$ square rods the area of an ellipsis in the given proportion; then $9,4248 : 3 \times 3 :: 160 : 152,7884$ — the square of the shorter diameter; then $\sqrt{152,7884} = 12,32 +$ rods the conjugate diameter required, and $3 : 12,32 :: 4 : 16,43$ —, the transverse.

31. $231 \times 10 = 2310$ cubic inches the capacity of the hollow globe; then $2310 \div ,5236 = 4411,764 +$ the cube of the diameter of the hollow globe; then $\sqrt[3]{4411,764} = 16,4 +$ inches the diameter of the hollow globe; then $16,4 + ,5 = 16,9$ the diameter including the thickness of the glass; then $16,9 \times 16,9 \times 16,9 \times ,5236 = 2527,3171924$ cubic inches, the solidity of the hollow globe including the glass; then $2527,3171924 - 2310 = 217,3171924$ cubic inches the solidity of the glass; then $217,3171924 \div 5236 = 415,0453 +$ the cube of the diameter of the solid globe required; then $\sqrt[3]{415,0453} = 7,45 +$ inches, the diameter required.

32. $60 \times 72 = 4320$ the product of the two diameters, and $72 - 60 = 12$ the difference, and $12 \times 12 \div 3 = 48$, one-third of the square of the difference; then $4320 + 48 = 4368$; then $4368 \times ,7854 = 3430,6272$ square inches the area of the surface in cylindric form, or the mean area between the two ends of the cistern; $30 \times 31,5 \times 231 = 218295$ cubic inches the solidity of the cistern; then $218295 \div 3430,6272 = 63,6 +$ inches $= 5,3$ feet, the depth required.

33. $100 \times 100 = 10000$ the square of the length of the tree, and $40 \times 40 = 1600$ the square of the base, and $10000 + 1600 = 11600$ the sum of those squares,

and $100 \times 2 = 200$, twice the length of the tree; then $1160 \div 200 = 58$ feet, the length of the broken piece.

34. $5280 \times 8000 = 42240000$ the number of feet in the earth's diameter; then $42240000 + 5 \times 5 = 211200025$ the square of the distance where a level from the eye will strike the water; then $\sqrt{211200025} = 14532,378 +$ feet the distance, and $14532,378 \div 5280 = 2,752 +$ miles the distance where a level from the eye strikes the water; then $8000 + 2,5 \times 2,5 = 20006,25$ the square of the distance where a level from the top of the mountain strikes the water; then $\sqrt{20006,25} = 141,443 +$ miles the distance; then $141,443 + 2,752 = 144,195$ miles, the whole distance required.

35. 3,1416 the area of the surface of a globe whose diameter is 1; then $3,1416 \times \frac{2}{3} = 2,0944$ the area of the surface required; then $2,0944 \div ,5236 = 4$ inches, the diameter required.

36. $4 \times 4 \times 4 \times ,5236 = 33,5104$ cubic inches the solidity of one ball, and $20 \times 112 = 2240$ *lbs.* one ton; then $2240 \times 16 = 35840$ ounces = one ton; then 7425 the specific gravity of cast iron : 1728 :: 35840 : 8341 — cubic inches contained in one ton of cast iron; then $8341 \div 33,5104 = 249$ — nearly, the number of balls required.

37. $50 + 60 + 40 = 150$ the sum of the three given sides; then $150 \div 2 = 75$ rods the half sum, and $75 - 50 = 25$, and $75 - 60 = 15$, and $75 - 40 = 35$; then $75 \times 25 \times 15 \times 35 = 984375$ the square of the area; then $\sqrt{984375} = 992,2$ square rods $= 6,2 +$ acres, the area required.

38. $6 \times 6 \times 2 = 72$, and $\sqrt{72} = 8,5$ —, and $8,5 + 6 = 14,5$ chains the side of the square; then $14,5 \times 14,5 = 210,25$ square chains $= 21,025$ acres, the area required.

39. $20 \times 20 = 400$ the square of the given side, and $400 \times 400 = 160000$, the fourth power of the given side, and $96 \times 96 \times 16 + 160000 = 307456$, and $307456 \times 7 \times 7 = 15065344$, and $20 \times 20 \times 7 \times 7 =$

19600, and 20 × 20 = 400; then 19600 + 400 = 20000, and 20000 ÷ 2 = 10000, and 10000 × 10000 = 100000000, and 100000000 — 15065344 = 84934656; then $\sqrt{84934656}$ = 9216, and 10000 — 9216 = 784; then $\sqrt{784}$ = 28, and 28 ÷ 7 = 4, which multiplied by the given proportion becomes 4 × 3 = 12 chains the shorter side, and 4 × 4 = 16 the longer.

40. 1,5 × 1,5 × 1,5 × ,5236 = 1,76715 cubic feet, the solidity, and 6 × 6 × 5 × ,5236 = 94,248 the solidity of the proportional spheroid; then 94,248 : 6 × 6 × 6 :: 1,76715 : 7,05 the cube of the greater diameter; then $\sqrt[3]{7,05}$ = 1,918 — feet the greater diameter required, and 1,918 × 5 ÷ 6 = 1,598 + feet, the less.

41. 18 × 18 × 15 × ,5236 = 2544,696 cubic inches the solidity of the oblate, and 15 × 15 × 18 × ,5236 = 2120,58 cubic inches the solidity of the prolate spheroid; then 2544,696 — 2120,58 = 424,116 cubic inches, the difference required.

42. 4 × 5 = 20, and 1 × 1 ÷ 3 = $\frac{1}{3}$; then 20 + $\frac{1}{3}$ = 20,333 + the product of the top and bottom diameters with $\frac{1}{3}$ of the square of the difference added; then 20,333 × 6 × ,7854 = 95,82 + cubic feet the solidity of the cistern, and 95,82 ÷ 2 = 47,91 cubic feet the solidity of the water, and 5 — 4 = 1 the difference of the two diameters of the cistern; then 1 : 5 :: 6 : 30 feet, the altitude of its taper continued until it came to a point; then 5 × 5 × ,7854 × 10 = 196,35 cubic feet the solidity if it were a cone whose diameter at the base was equal to the diameter of the base of the cistern; then 196,35 — 47,91 = 148,44 cubic feet the solidity left at the vertex of the cone; then 196,35 : 30 × 30 × 30 :: 148,44 : 20411,912 + the cube of the altitude above the water; then $\sqrt[3]{20411,912}$ = 27,324 + feet the altitude above the water; then 30 — 27,324 = 2,676 — feet, the depth of the water in the cistern.

43. 24 × 10 = 240 square chains the area; then 240 ÷ ,433013 = 554,256 — the square of the side; then $\sqrt{554,256}$ = 23,5426 + chains, the length of the side required.

44. $2150,4252 \times 60 = 129025,512$ cubic inches the solidity of the box; then $\sqrt[3]{129025,512} = 50,53 +$ inches, the length of the side required.

45. $160 \div 2 = 80$ square rods the area of the garden, and $80 \div ,7854 = 101,8576 +$ the square of the diameter of the garden; then $\sqrt{101,8576} = 10,09 +$ rods the diameter of the garden, and $80 \div 4 = 20$, and $80 + 20 = 100$ square rods the area of the garden and walk; then $100 \div ,7854 = 127,3236 +$ the square of the diameter including the walk; then $\sqrt{127,3236} = 11,28 +$ rods the diameter including the walk; then $11,28 - 10,09 = 1,19$ rods the difference of diameters; then $1,19 \div 2 = ,595$ rods, the width of the walk required.

46. $42 \times 2 = 84$ feet the length of the sides, and $2 \times 2 = 4$ feet twice the thickness of the wall; then $26 - 4 = 22$ feet the length of each end; then $22 \times 2 = 44$ feet the length of both ends; then $84 + 44 = 128$ feet the whole length of the wall; then $128 \times 7 \times 2 \div 16,5 = 108,6 +$ perches, the answer required.

47. $20 \times 112 \times 2 \times 16 = 71680$ avoirdupois ounces the weight of the anchor; then as 7645 ounces the specific gravity : 1 :: 71680 : 9,376 + cubic feet, the solidity required.

48. $3 \times 12 = 36$ inches the diameter of the globe; then $36 \times 3 \div 11 = 9,9$ inches, and $36 - 9,9 = 26,1$ inches the perpendicular altitude of the largest equangular pyramid that can be formed from the globe given; then $26,1 \times 9,9 \times 4 = 1033,56$ the square of its side; $1033,56 \times \sqrt{1033,56} \times ,11785 \div 1728 = 2,2083 +$ cubic feet the solidity of the pyramid, and $3 \times 3 \times 3 \times ,5236 = 14,1372$ cubic feet the solidity of the globe; then $14,1372 - 2,2083 = 11,9289$ cubic feet, the solidity of the chips required.

49. $3,1416 \times 60 = 188,496$ chains the length of the periphery of the whole circle; then 360 : 188,496 :: 24 : 12,5664 chains, the length of the arc required.

50. $160 \times 5 = 800$ square rods the sum of the length by the breadth, and $120 \div 2 = 60$ rods the sum of the

length and breadth of the parallelogram; then $60 \div 2 = 30$ rods the half sum, and $30 \times 30 = 900$ the square of the half sum, and $900 - 800 = 100$ the square of half the difference of the two unknown sides; then $\sqrt{100} = 10$ rods half the difference, and $30 + 10 = 40$ rods the longer side, and $30 - 10 = 20$ rods, the shorter.

51. $20 \times 20 \times 4{,}828427 \div 9 = 214{,}5967 +$ square yards, the area required.

52. $282 \times 25 = 7050$ cubic inches the capacity of the kettle; $5 \times 4 = 20$ the product of the top and bottom diameters, and $5 - 4 = 1$; then $1 \times 1 \div 3 = \frac{1}{3}$, and $20 + \frac{1}{3} = \frac{61}{3}$; then $\frac{61}{3} \times 6 \times {,}7854 = 95{,}8188$ cubic inches the solidity of a kettle corresponding with the given proportions; $95{,}8188 : 6 \times 6 \times 6 :: 7050 : 15892{,}4 +$, the cube of the depth; then $\sqrt[3]{15892{,}4} = 25{,}14 +$ inches the depth of the kettle; then $6 : 25{,}14 :: 5 : 20{,}95$ inches the top diameter, and $6 : 25{,}14 :: 4 : 16{,}76$ inches, the bottom required.

53. $90 \div 60 = 1{,}5$, twice the square of the semidiameter; then $1{,}5 \div 2 = {,}75$ the square of the semidiameter; then $\sqrt{{,}75} = {,}866 +$ the semidiameter; then ${,}866 \times 2 = 1{,}732$ feet, the diameter of the cylinder required.

54. $8 + 7 + 5 = 20$ the sum of the three proportional sides, and $20 \div 2 = 10$ the half sum; then $10 - 8 = 2$, and $10 - 7 = 3$, and $10 - 5 = 5$; then $10 \times 2 \times 3 \times 5 = 300$ the square of the area of the proportional sides; then $\sqrt{300} = 17{,}32 +$ the proportional area, and $60 \times 10 = 600$ square chains the area of the triangle whose sides are required; then $17{,}32 : 5 \times 5 :: 600 : 866$ the square of the shorter side; then $\sqrt{866} = 29{,}428 -$ chains the shorter side; then $5 : 29{,}428 :: 8 : 47{,}0848$ chains the longer side, and $5 : 29{,}428 :: 7 : 41{,}1992$ chains, the length of the other.

55. $60 + 50 + 40 \div 2 = 75$ the half sum of the three sides; then $75 - 60 = 15$, and $75 - 50 = 25$, and $75 - 40 = 35$; then $75 \times 15 \times 25 \times 35 = 984375$ the square of the area of the triangle; then $\sqrt{984375} = 992{,}157$ square chains the area; then $992{,}157 \times \frac{2}{3} =$

661,438 square chains the area left at the vertical angle after the first division; then 992,157 : 60 × 60 : : 661-,438 : 2400 the square of the base left after the first division; then $\sqrt{2400} = 49$ — the length; 992,157 ÷ 3 = 330,719 square chains left at the vertical angle after the second division; then 992,157 : 60 × 60 : : 330,719 : 1200 the square of the base after the second division; then $\sqrt{1200} = 34,636$ — chains, the length of the base after the second division.

56. 3 + 2 = 5 the sum of the proportion, and 5 — 3 = 2 the proportional part left at the vertical angle; then 5 : 30 × 30 : : 2 : 360 the square of the side left at the vertical angle after the greater part is cut off; then $\sqrt{360} = 19$ — chains the length of each of the equal sides left at the vertical angle, and 5 : 20 × 20 : : 2 : 160 the square of the base of the less part; then $\sqrt{160} = 12,65$ — chains, the length of the base of the less part.

57. 60 × 60 = 3600 the square of the transverse diameter, and 40 × 40 = 1600 the square of the conjugate, and 3600 + 1600 = 5200 the sum of their squares; then 5200 × 4 = 20800, and $\sqrt{20800} = 144,222$ +, and 40 ÷ 3 = 13,333 + the third of the conjugate; then 144,222 + 13,333 + = 157,555 chains, the length of the periphery required.

58. 40 × 40 = 1600 the square of the base, and 30 × 30 = 900 the square of the perpendicular, and 1600 + 900 = 2500 the square of the hypotenuse, and $\sqrt{2500}$ = 50 chains the hypotenuse, and 40 + 30 = 70 the sum of the base and perpendicular; then 70 — 50 = 20 chains, the diameter of the inscribed circle required.

59. 124 × 124 = 15376, and 42 × 42 = 1764, and 15376 — 1764 = 13612, and 124 × 2 = 248; then 13612 ÷ 248 = 54,8871 — chains the perpendicular, and 124 — 54,8871 = 69,1129 chains, the third side.

60. 18 ÷ 2 = 9 inches the semidiameter of the circle, and 9 × 9 = 81 its square, and 11 ÷ 2 = 5,5 inches half the chord; 5,5 × 5,5 = 30,25 the square of half the chord; then 81 — 30,25 = 50,75, and $\sqrt{50,75} = 7,1239$ + inches the distance between the chord and centre of

the circle; then $9 - 7,1239 = 1,8771$ inches, the versed sine required.

61. $27 \div \sqrt{3} = 15,59$ chains the length of half the side; then $15,59 \times 2 = 31,18$ chains the length of the side; and $15,59 \times 15,59 \times ,433013 = 105,2429 +$ square chains the area, which divided by $10 = 10,52429$ acres, the area required.

62. $16 \times 16 = 256$, and $256 \times 3 = 768$ the square of the diameter of the globe; then $\sqrt{768} = 27,7 +$ inches, the diameter required.

63. $16 \div 2 = 8$ inches the semidiameter, and $8 \times 8 \times 2 = 128$, twice the square of the semidiameter, and $36 \times 144 = 5184$; then $5184 \div 128 = 40,5$ feet, the length required.

64. Suppose the side of the square to be one chain; then 4 chains the perimeter, and 1 square chain the area; then $\frac{1}{10}$ = area in acres, which is equal to the perimeter; then 4 the perimeter divided by $\frac{1}{10} = 40$ chains, the length of the side required.

65. $1 - ,7854 = ,2146$, and $4 \times 10 = 40$ chains the area in the corners; then $,2146 : 1 \times 1 :: 40 : 186,776242 +$, the square of the side; then $\sqrt{186,776242} + = 13.666 +$ chains, the length of the side required.

66. $18 \times 18 \times 18 \times 2 \div 3 = 3888$ the cube of the altitude left after the first section is cut off; then $\sqrt[3]{3888} = 15,724 +$ feet the altitude left; then $18 - 15,724 = 2,276$ feet the height of the thickest section, and $18 \times 18 \times 18 \div 3 = 1944$ the cube of the altitude left after the second section is cut off; then $\sqrt[3]{1944} = 12,48$ feet the altitude of the part left at the vertex; then $15,724 - 12,48 = 3,244$ feet, the height of the middle section.

67. $70 \times 70 = 4900$, and $50 \times 50 = 2500$; then $4900 - 2500 = 2400$, and $2500 - 2400 = 100$ the square of the difference; then $\sqrt{100} = 10$ the difference between the base and perpendicular of the triangle; then $10 \div 2 = 5$ rods half the difference, and $70 \div 2 = 35$ the half sum; then $35 + 5 = 40$ rods the length of the base, and $35 - 5 = 30$, the perpendicular required.

68. $3 \times 4 = 12$ the product of the two diameters,

and 4 — 3 = 1 the difference, and the square of the difference is also 1; then 1 ÷ 3 = ,333; then 12,333 + × 5 × ,7854 = 48,4316910 cubic inches the solidity of the given proportion, and 31,5 × 30 × 231 = 218295 cubic inches the solidity of the cistern; then 48,431691 : 5 × 5 × 5 : : 218295 : 563409,5 the cube of the length, the cube root of which will be found = 82,6 — inches the length of the cistern = 6,883 + feet; then 5 : 6,883 : : 4 : 5,5064 feet the bottom diameter, and 5 : 6,883 : : 3 : 4,1298 feet, the top diameter.

69. 20 × 45 = 900, and $\sqrt{900}$ = 30 chains the mean proportional between the greater and the less; then 30 × 2 + 30 = 90, and 90 ÷ $\sqrt{3}$ = 51,96 + chains the side of the equilateral triangle; then 51,96 × 51,96 × ,433013 = 1169,066 + square chains, which divided by 10 = 116,9066 acres, the area required.

70. 1 — ,5236 = ,4764 : 1 × 1 × 1 : : 2150,4252 : 4513,9 the cube of the side; then $\sqrt[3]{4513{,}9}$ = 16,526 + inches, the side of the cube required.

71. 64 × 64 — 40 × 40 = 2496, and 64 × 2 = 128, and 2496 ÷ 128 = 19,5 chains the breadth of the parallelogram; then 19,5 × 40 = 780 square chains the area = 78 acres, the area required.

72. 40 × 40 = 1600, and 24 × 24 = 576; then 1600 — 576 = 1024, and $\sqrt{1024}$ = 32 feet the distance from the foot of the ladder to the buildings, and 16 × 16 = 256; then 1600 — 256 = 36,666 + = 36 feet 8 inches; then 36 feet 8 inches + 32 feet = 68 feet 8 inches, the breadth of the street.

73. 30 × 30 = 900 the square of the given side, and 12 × 12 = 144 the square of the perpendicular; then 900 — 144 = 756, and $\sqrt{756}$ = 27,5 —; then 27,5 + 30 = 57,5, and 57,5 × 57,5 = 3306,25, and 144 + 3306,25 = 3450,25 the square of the diagonal required; then $\sqrt{3450,25}$ = 58,739, the longer diagonal of the rhombus.

74. 24 ÷ 2 = 12 chains, half the longer diagonal, and 12 × 12 = 144 its square, and 18 ÷ 2 = 9 chains, half the shorter diagonal, and 9 × 9 = 81 its square;

then 81 + 144 = 225; then √225 = 15 chains, the length of the side required.

75. 42 × 36 × ,7854 ÷ 2 = 593,7624 square rods the area of the triangle whose side is required; then 593,7624 ÷ ,433013 = 1371,2345 + the square of the side; then √1371,2345 = 37,03 rods, the length of the side required.

76. 50 — 30 ÷ 2 = 10 rods the versed sine of each segment made by the intercepted arcs of the circles; then 10 ÷ 50 = ,2 the tabular versed sine, and ,111823 its corresponding area; then ,111823 × 50 × 50 = 279,5575 square rods the area of each segment; then 279,5575 × 2 = 558,715 square rods included between the intercepted arcs, and 50 × 50 × ,7854 × 2 = 3927 square rods the area of both circles; then 3927 — 558-,715 = 3368,285 square rods the area enclosed by the peripheries of both circles = to 21,0518 — acres.

77. 100 × 60 = 6000 square feet the area of the garden, and 6000 ÷ 4 = 1500 square feet the area of the walk, and 100 + 60 ÷ 2 = 80 feet the half sum of the two sides, and 80 × 80 = 6400 the square of the half sum, and 6400 — 1500 = 4900, and √4900 = 70; then 80 — 70 = 10 feet, the width of the walk.

78. 60 ÷ 3,1416 = 19 + feet the diameter, and 19 ÷ 2 = 9,5, and 9,5 × 9,5 = 90,25 its square, and 120 × 120 = 14400 the square of the perpendicular; then 14400 + 90,25 = 120,41 — feet the length of the slope side; then 120,41 × 60 ÷ 2 = 361,23 square feet the area of the convex surface of the conical spire; then 361,23 × ,08 ÷ 9 = $3,21 +, the expense required.

79. 1 ÷ 40 = ,025 of an inch the diameter of the wire; then ,025 × ,025 × ,7854 = ,00490875 of a square inch the area of the end of the wire; then 1728 ÷ ,00490875 = 352024,446 + inches the length of the wire = to 5,556 + miles.

80. 3 — 1 = 2; then 2 : 18 : : 3 : 27 feet the length of the stick if it were a pyramid; then 3 × 3 = 9 square feet the area of its base, and 27 ÷ 3 = 9 feet, one-third of the perpendicular altitude; then 9 × 9 = 81 cubic

feet the solidity if it were a pyramid, and $27 - 18 = 9$ feet the altitude added to the small end of the frustrum to make it a pyramid, and 1 foot its base; then $1 \times 1 \times 3 = 3$ cubic feet the solidity required to make the frustrum a pyramid; then $81 - 3 = 78$ cubic feet the solidity of the frustrum, and $78 \div 3 = 26$ cubic feet each man's share; then $81 : 27 \times 27 \times 27 :: 81 - 26 : 13365$ the cube of the altitude of the pyramid left at the vertex after the first share is cut off from its base; then $\sqrt[3]{13365} = 23{,}73 +$ feet the altitude left at the vertex; then $27 - 23{,}73 = 3{,}27$ feet the altitude of the thickest section, and $81 : 27 \times 27 \times 27 :: 29 : 7047$, cube of the altitude left after the second share is taken off; then $\sqrt[3]{7047} = 19{,}17 +$ feet the altitude; then $23{,}73 - 19{,}17 = 4{,}56$ feet the altitude of the middle section, and $19{,}17 - 9 = 10{,}17$ feet the altitude at the small end; then the length of the shares is 10,17 —, 4,56, and 3,27 feet.

81. $2150{,}4252 \div 2 = 1075{,}2126$ cubic inches the solidity of half a bushel, and $,7854 \times 8 = 6{,}2832$, and $1075{,}2126 \div 6{,}2832 = 155{,}05 +$ the square of the diameter; then $\sqrt{155{,}05} = 12{,}452 -$ inches, the diameter required.

82. $5 \times 5 \times ,7854 \times 2 = 39{,}27$ cubic inches the capacity of the glass, and $5{,}148 + 2 = 7{,}148$ inches the distance from the vertex to the top of the globe; then $7{,}148 - 6 = 1{,}148$ inches the axis of the globe above the glass; then $4 - 1{,}148 = 2{,}852$ inches the axis within the glass; then $2{,}852 \times 1{,}148 = 3{,}274096$ the square of the semidiameter of the globe level with the top of the glass; then $3{,}274096 \times 3 + 1{,}148 \times 1{,}148 \times ,5236 \times 1{,}148 = 6{,}696 +$ cubic inches the solidity of the segment above the glass, and $4 \times 4 \times 4 \times ,5236 = 33{,}5104$ cubic inches the solidity of the globe; then $33{,}5104 - 6{,}696 = 26{,}8144$ cubic inches the solidity of the globe within the glass, and consequently the solidity of the water displaced.

83. $5 - 4 = 1$; then $1 : 12 :: 5 : 60$ feet, the perpendicular altitude of a cone whose base is 5 feet and dimi-

nishing one foot in twelve, and $5 \div 2 = 2{,}5$ feet half the diameter, and $60 \times 60 = 3600$ the square of the altitude, and $2{,}5 \times 2{,}5 = 6{,}25$ the square of the semidiameter of the base; then $3600 + 6{,}25 = 3606{,}25$ the square of the slope of the cone, and $\sqrt{3606{,}25} = 60{,}05$ + feet the slope of the cone, and consequently the semidiameter of the circle required; then $60{,}05 \times 2 = 120{,}1$ feet the diameter of the circle made by the greater wheel, and $12 \times 2 = 24$, and $120{,}1 - 24 = 96{,}1$ feet the diameter made by the less, and $120{,}1 \times 120{,}1 = 14424{,}01$ the square of the diameter of the circle made by the greater wheel, and $96{,}1 \times 96{,}1 = 9235{,}21$ the square of the diameter of the circle made by the less wheel, and $14424{,}01 - 9235{,}21 = 5188{,}8$ the difference; then $5188{,}8 \times {,}7854 = 4075{,}28352$ square feet, the area of the ring required.

84. $8 + 10 \div 2 = 9$ chains the average width, and $9 \times 20 = 180$ square chains the area of the field; then $180 \div 10 = 18$ acres the whole area, and $10 - 8 = 2$; then $2 : 20 :: 8 : 80$ chains the length continued to the small end to form the whole to a triangle, and $80 + 20 = 100$ chains the whole length of the base of the triangle, and by the question 10 chains the perpendicular; then $100 \times 10 \div 2 = 500$ square chains the area of said triangle; $18 - 8 = 10$ acres, or 100 square chains, to be left at the perpendicular of the triangle; then $500 - 100 = 400$ square chains to be left at the vertical angle; then $500 : 100 \times 100 :: 400 : 8000$ the square of the base; then $\sqrt{8000} = 89{,}4427$ + chains the distance from the vertical angle to the place where the line of separation must be drawn; then $89{,}4427 - 80 = 9{,}4427$ + chains, the distance from the southwest corner, and $80 : 8 :: 89{,}4427 : 8{,}94427$ chains, the length of the division line.

85. $4 : 12 \times 12 \times 12 :: 3 : 1296$ the cube of the altitude; then $\sqrt[3]{1296} = 10{,}9$ + feet the perpendicular altitude left at the vertex; then $12 : 4 :: 10{,}9 : 3{,}633$ + feet the diameter where the section is cut off; then $3{,}633 \div 2 = 1{,}816$ feet the semidiameter. and $1{,}816 \times$

1,816 = 3,297856 its square, and 10,9 × 10,9 = 118,81 the square of the perpendicular altitude, and 118,81 + 3,297856 = 122,1078 the square of the length on the slope; then √122,1078 = 11,05 feet, the length required.

86. Between the equator and the poles are 90 degrees; then 45 ÷ 90 = ,50, and √50 = ,70710678, and ,70710678 × 8000 = 5656,85424 miles, the diameter required.

87. 16 — 15,5 = ,5; then 5 : 20 : : 16 : 640 feet the distance from the second stake to the opposite bank of the river; then 640 — 20 = 620 feet, the breadth of the stream.

88. Suppose the less number is 10; then 25 — 10 = 15, one-third of the other two; then 15 × 3 = 45 the sum of the two greater. Again suppose the less of the two greater is 19; then 45 — 19 = 26; the three numbers would then be 10, 19, and 26, and 10 + 26 ÷ 4 = 9, and 19 + 9 = 28, which by the condition of the question should be 25; then 28 — 25 = 3 the error. Again, suppose the less of the two greater to be 11; then 45 — 11 = 34 the greater; then the three numbers would be 10, 11, 34; then 34 + 10 ÷ 4 = 11, and 11 + 11 = 22, which by the question should be 25; then 25 — 22 = 3 the error. The errors being alike in quantity but not in quality, the half sum of the two suppositions must be the number sought; then 19 + 11 ÷ 2 = 15 the less of the two greater, if the smaller be 10, as was supposed; then 45 — 15 = 30 the greater; then the three numbers would be 10, 15, and 30, two of which answer the conditions of the question; then 10 + 15 ÷ 5 = 5, and 30 + 5 = 35, which by the question should be 25; therefore 35 — 25 = 10 the error. Again, suppose the less of the three to be 8; then 25 — 8 = 17, one-third of the other two; then 17 × 3 = 51 the sum of the two greater; then suppose the less of the two greater to be 19; then 51 — 19 = 32, and 32 + 8 = 40, and 40 ÷ 4 = 10, and 10 + 19 = 29, which by the conditions of the question should be 25; then

$29 - 25 = 4$ the error. Again, suppose the less of the two greater numbers to be 15; then $51 - 15 = 36$, and $36 + 8 = 44$, and $44 \div 4 = 11$, and $11 + 15 = 26$, which by the conditions should be 25; then $26 - 25 = 1$ the error; then by multiplying the positions by the errors, that is, $19 \times 1 = 19$, and $15 \times 4 = 60$, and $60 - 19 = 41$, and $41 \div 3 = \frac{41}{3}$ the second number if the first be 8, and $51 - \frac{41}{3} = \frac{112}{3}$ the greater; then $\frac{41}{3} + 8 \div 5 = \frac{13}{3}$, which added to $\frac{112}{3} = \frac{125}{3}$; then $\frac{125}{3} - 25 = \frac{50}{3}$ the error; then the first position $10 \times \frac{50}{3} = \frac{500}{3}$ and the second position $8 \times 10 = 80$; then because the errors are alike, $\frac{500}{3} - 80 = \frac{260}{3}$ the difference of the products, and $\frac{50}{3} - 10 = \frac{20}{3}$; then $\frac{260}{3} \div \frac{20}{3} = 13$ the less number, and the second proportional number; $15 \times \frac{50}{3} = \frac{750}{3}$, and $\frac{41}{3} \times 10 = \frac{410}{3}$; then $\frac{750}{3} - \frac{410}{3} = \frac{340}{3}$; then $\frac{340}{3} \div \frac{20}{3} = 17$ the less of the two greater, and $17 + 13 \div 5 = 6$, and $25 - 6 = 19$ the greater; then the respective distances from the point within the equilateral triangle to the angles are 13, 17, and 19 chains, and $13 \times 19 = 247$, and $\sqrt{247} = 15{,}716$ + chains the mean proportional between the greater and the less; then $15{,}716 \times 2 + 17 \div \sqrt{3} = 27{,}96$ + chains, the length of the side required.

89. $5 \times 12 = 60$ inches the diameter of the stone; then $60 \times 60 \times 3 \div 4 = 2700$ the square of the diameter left after the first has ground off his share; then $\sqrt{2700} = 52$ — nearly, the diameter left; then $60 - 52 = 8$ inches of the diameter the first share, and $2700 \times 2 \div 3 = 1800$ the square of the diameter left after the second has ground off his share; then $\sqrt{1800} = 42{,}4264$ + inches the diameter left, and $52 - 42{,}4264 = 9{,}5736$ inches the diameter of the second share, and $1800 \div 2 = 900$ the square of the diameter left after the third has ground his share; then $\sqrt{900} = 30$ inches the diameter of the fourth share; then $42{,}4264 - 30 = 12{,}4264$ inches, the third share.

90. Suppose the diameter of each of the equal circles to be 10 chains; then $10 \times 10 \times 7854 = 78{,}54$ square chains the area of each circle, and as their centres are

equally distant from each other, so that lines drawn from the centres to the centres of others form an equilateral triangle whose sides are each equal to the diameter of the equal circles; then $10 \times 10 \times ,433013 =$ 43,3013 square chains contained in the triangle, and as each of the angles contains 60 degrees, then $360 : 78,54 :: 60 : 13,09$ square chains the area of the sector of each circle; then $13,09 \times 3 = 39,27$ square chains the area of the three sectors, and $43,3013 - 39,27 = 4,0313$ square chains the area contained between the peripheries; then $4,0313 : 10 \times 10 :: 20$ square chains the area given: 496,12 — the square of the diameter of each of the equal circles to be enclosed by the fourth; then $\sqrt{496,12} = 22,2706$ chains the diameters, and likewise the length of the side of an equilateral triangle made by the centres of the three circles; then 22,2706 $\times \sqrt{3} = 38,57379$ chains, three times the distance from the centre of the triangle to the centre of each circle; then $38,57379 \div 3 = 12,85793 +$ chains the distance, and $22,2706 \div 2 = 11,1353$ chains the semidiameter of each of the equal circles; then $11,1353 + 12,85793$ $= 23,99323$ chains the semidiameter of the circle enclosing the other three; then $23,99323 \times 2 = 47,98646$ chains, the diameter required.

91. $2,5 : 2,4 + 2 :: 2,4 - 2 : ,704$ the difference of the segments of the base made by a perpendicular let fall from the opposite angle to the base; then $704 \div 2$ $= ,352$, half the difference, and $2,5 = 1,25$ the half sum; then $1,25 - ,352 = ,898$ of a mile the shorter segment; then $2 \times 2 = 4$ the square of the shorter side, and $,898 \times 898 = ,806404$ the square of the shorter segment; then $4 - 806404 = 3,193596$ the square of the perpendicular; then $\sqrt{3,193596} = 1,787$ + miles the perpendicular; then $1,787 : 2,4 :: 2 :$ 2,686 miles the diameter of a circle whose periphery would pass through each of the seats; then $2,686 =$ 1,343 miles, the distance required.

92. $40 : 30 + 20 :: 30 - 20 : 12,5$ the difference of the segments; then $12,5 \div 2 = 6,25$ the half difference,

and 40 ÷ 2 = 20 rods the half sum; then 20 — 6,25 = 13,75 rods the shorter segment, and 13,75 × 13,75 = 189,0625 its square, and 20 × 20 = 400 the square of the shortest side of the triangle; then 400 — 189,0625 = 210,9375 the square of the perpendicular which divides it into two right angles; then √210,9375 = 14,5236 + rods the perpendicular; then 14,5236 : 30 : : 20 : 41,312 + rods, the diameter required.

93. 21 ÷ 2 = 10,5, half the diagonal; then 10,5 × 10,5 = 110,25 the square of half the diagonal, and 84 × 2 = 168 double the area, and 168 ÷ 21 = 8 chains the perpendicular, and 110,25 + 8 + 8 × 8 × 4 = 729 the square of the sum of the two required sides; then √729 = 27 chains the sum of the two sides, and 85 + 50 = 135 the sum of the proportion given; then 135 : 50 : : 27 : 10 chains the shorter side, and 27 — 10 = 17 chains, the longer.

94. Suppose the diameter of the circle to be 50 chains; then 50 × 50 × ,7854 × 20 ÷ 10 = 3927 dollars its value at 20 dollars per acre, and 50 × 66 × 12 × 3,1416 = 124407,36 inches the circumference of the supposed circle; then 124407,36 × 2 ÷ 3 = 82938,24 dollars the number which would encompass the supposed circle; then 3927 : 50 : : 82938,24 : 1056 chains the diameter of the circle required, and 1056 × 3,1416 × 66 × 12 × 2 ÷ 3 = 1751655,6288 dollars, the price it cost.

95. 16 × 16 × 2,598076 = 665,1074 + square feet the area, and 16 × 6 ÷ 2 = 48 feet, half the perimeter; then 665,1074 ÷ 48 = 13,8564 feet, the length of the perpendicular required.

96. 5 × 4 + 2 = 22 roods, and 22 × 40 + 25 = 905 square perches the area of the field; then 905 ÷ 1,720477 = 526,0221 + the square; √526,0221 + = 22,935 + perches, the length of the side required.

97. 8000 × 8000 × 8000 = 512000000000 the cube of the diameter of the earth; 2180 × 2180 × 2180 = 10360232000 the cube of the diameter of the moon; then 512000000000 : 240000 : : 10360232000 : 4856-,35875 miles the distance from the earth's centre; then

4856,35875 — 4000 miles the semidiameter of the earth = 856,35875 miles above the surface.

98. Suppose the equal circles to be 10 chains in diameter; then the centres are likewise 10 chains asunder; and consequently if lines be drawn from centre to centre, those lines form an equilateral triangle whose sides are 10 chains; then $10 \times \sqrt{3} \div 3 = 5{,}7735$ chains from the centre of the supposed figure to the centre of each of the supposed circles; then $5{,}7735 + 5 = 10{,}7735$ chains the semidiameter of a circle which will just encompass the three supposed circles; then $10{,}7735 \times 2 = 21{,}547$ chains the diameter, and $21{,}547 \times 21{,}547 \times {,}7854 = 364{,}64$ square chains the area of a circle encompassing the three supposed circles, and $100 \times 100 \times {,}7854 = 7854$ square chains the area of the circle whose diameter is given; then $364{,}64 : 10 \times 10 :: 7854 : 2153{,}905221$ the square of the diameter of each of the required circles; then $2153{,}905221 \times {,}7854 \div 10 = 169{,}167716 +$ acres the portion of each of the sons; then $169{,}167716 \times 3 = 507{,}503148$ acres their share, and $7854 \div 10 = 785{,}4$ acres the area contained in the whole circle; then $785{,}4 - 507{,}503148 = 277{,}896852$ acres, the widow's share.

99. $250 \times 2 = 500$, and $40 \div 2 = 20$ chains half the length of the base, and $20 \times 20 \times 2 = 800$, twice the square of half the base; then $800 + 500 = 1300$ the sum of their squares = to the sum of the squares of the other two sides, and $3 \times 3 = 9$, and $2 \times 2 = 4$; then $9 + 4 = 13$ the sum of the squares of the given proportion; then $13 : 1300 :: 4 : 400$ the square of the shorter side; then $\sqrt{400} = 20$ chains the shorter side, and $13 : 1300 :: 9 : 900$ the square of the longer side; then $\sqrt{900} = 30$ chains, the longer side.

100. Suppose the side of the nonagon to be 10 chains; then $10 \times 10 \times 6{,}181824 = 618{,}1824$ square chains the area of the nonagon whose side is 10 chains; then $9 \times 10 \div 2 = 45$ chains, half its perimeter; then $618{,}1824 \div 45 = 13{,}7374 -$ chains, the length of a perpendicular let fall from the centre of the nonagon to the middle

of one of its sides, the side of the pentagon being equal to that of the nonagon; therefore $10 \times 10 \times 1{,}720477 = 172{,}0477$ square chains the area of each pentagon; then $10 \times 5 \div 2 = 25$ chains, half the perimeter, and $172{,}0477 \div 25 = 6{,}4819 +$ chains the length of a perpendicular let fall from the centre of one of the pentagons to the middle of one of its sides; then $6{,}4819 + 13{,}7374 = 20{,}2193$ chains the distance from the centre of the nonagon to the centre of one of the pentagons; then $6{,}4819 \times 6{,}4819 = 42{,}01502761$ the square of the perpendicular of one of the pentagons, and $10 \div 2 = 5$ chains, half the side of the supposed pentagon; then $5 \times 5 = 25$ its square, and $42{,}01502761 + 25 = 67{,}01502761$ the square of the distance from the centre of one of the pentagons to each of the angles; then $\sqrt{67{,}01502761} = 8{,}18627$ chains the distance; then $20{,}2193 + 8{,}18627 = 28{,}40557$ chains the distance from the centre of the supposed nonagon to the periphery of a circle, and consequently the semidiameter of a circle encompassing the supposed nonagon and pentagons; then $28{,}40557 \times 2 = 56{,}81114$ chains its diameter; then $56{,}81114 \times 56{,}81114 \times {,}7854 = 2534{,}883$ — square chains the area of the circle; then $2534{,}883 : 10 \times 10 :: 7854 : 309{,}837$ — the square of the length of the side of the required nonagon, and likewise of the pentagons; then $309{,}837 \times {,}6181824 = 1915{,}3578 +$ square chains $= 191{,}53578$ acres the area of the nonagon, and therefore the son's share, and $1{,}720477 \times 309{,}837 = 533{,}0674 +$ square chains $= 53{,}30674$ acres each of the daughters' share, and $53{,}30674 \times 9 + 191{,}53578 = 671{,}29644$ acres belonging to the son and daughters; then $785{,}4 - 671{,}29644 = 114{,}10356$ acres, the widow's part.